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Application of the Concept of Fractal for the Stress Assessment of the Condition of an Object

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ABSTRACT

In the given paper application of fractal analysis to an estimation of the intense-deformed condition of a concrete slab with a hole-forming cores is observed. The new technique of an estimation of the strained condition of the specified concrete slab with use of the concept of a fractal is presented. The most widespread way of an estimation of a tension is connected with the use of a matrix of rigidity of system. In the paper the estimation of a course-of-value function of a measure which characterizes the loaded condition of a concrete slab is given. By means of a method of least squares and a method of the fastest descent, character of the intense-deformed condition of an object is assessed.

Keywords: Matrix numbers, Fractal dimension, Course-of-value function, Fractal analysis.

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1. PROBLEM STATEMENT

The most widespread way of an estimation of a tension is connected with the use of a matrix of rigidity of system. We present short survey of the given technique. The armature is connected with concrete by means of special binding elements (BE), admitting mutual shifts of armature concerning the concrete, simulating the grip of armature with concrete. Consider that x is a relative shear of armature. The accepted mathematical model allows connecting a finite element of concrete with a finite element of armature at various length of a linear finite element (FE) of armature. In the capacity of a variable quantity the length of linear finite element of armature is accepted. Length of the finite element of the rectilinear, not unbent, stressed and not stressed armature are equal to a size of triangular finite element of concrete, and communication of finite element is carried out in each knot of a grid along the length of armature. The arrangement of binding elements is defined by means of mathematical model, the design scheme of a beam and an operating field of numbers. In such each nodal point in which binding element is absent, in the field of numbers it is marked as 0. In a point in which binding element occurs, the number distinct from 0 is established. And, if binding element unites finite element of concrete with the bottom rectilinear stressed armature in the corresponding knot of an operating field of numbers it is established 1, if with the bent stressed armature, number 2, and, if with upper not stressed number 3 is established. Such field of numbers can be changed depending on a slope and length of the bent part of armature. When formulating a matrix of rigidity of a system the following fact is considered: presence of binding element at any knot is taken into account by means of correction and change of corresponding sections of a matrix of rigidity of a system. Relationship between forces in finite element's bonding and mutual displacements of armature and concrete (where F-force in bonding) at level of a surface of contact is calculated by the following formula:

$$F_i = k_i \Delta x_i \quad (1)$$

where k_i – rigidity of the horizontal bondings

Taking into account that stresses, at an end face of an element from the force of pre-stressing and act of external loading are equal, in armature to 0, and were corrected depending on length of a linear element, and then zero boundary conditions are accepted. The relationship between rigidities of horizontal and vertical bondings of bonding elements, when there are two yieldable bondings along co-ordinate axes, was defined by dependence: (2)

$$k_i^y = k_i^x \times \frac{F_i^y}{F_i^x} \cdot \mathrm{tg}\alpha \quad (2)$$

where F_i^y, F_i^x –forces directed along horizontal and vertical axes, α–slope of the bent armature. The problem of defining rigidities k_i^y, k_i^x, usually can be solved by a method of consecutive approximations. Originally at first approximation, rigidity $k_i^{y(1)}, k_i^{x(1)}$ is set and calculation, by definition of mutual displacements of armature and concrete on a vertical Δy_i and a horizontal Δx_i, is made. Using a condition (1), by the found values Δy_i and Δx_i, forces $F_i^{y(1)}, F_i^{x(1)}$ are defined, and then these values are substituted in expression (2) for obtaining values $k_i^{y(2)}, k_i^{x(2)}$ of a second approximation. Calculations are being repeated until values $\Delta x_i, \Delta y_i$, of the previous and subsequent calculations did not attain demanded accuracy. Additional anchoring of armature for the account of bending in a bend point was considered by increase k_i^y, k_i^x to the values providing compatibility of displacements on a vertical of knots of concrete and armature in the given point. The force from armature pre-stressing was put, as external constricting force, to extreme finite element at an armature end face. Stressing from external loading was applied to the knots of design scheme. For an estimation of an intense-deformed condition of a preliminary strained ferroconcrete beam, the block diagram for a program writing in language of a high level is made. In the capacity of initial data the geometrical sizes of beams, corresponding to development works, the data on its breakdown on finite elements, elastic characteristics of materials (concrete and armature), rigidity of bonding of each type of armature and concrete (modules of elasticity E of concrete and armature), are accepted. Besides, the initial information should contain numbers of knots of the application of external loading and magnitudes of external loading. Procedure of calculatation:

1) The beam is broken into finite elements of concrete and armature;
2) The operating field of numbers is compiled;
3) The local matrixes of rigidity, considering physico-mechanical and geometrical characteristics of finite elements, are calculated;
4) The global matrix of rigidity, in an aspect of coefficients of systems of the solvable linear algebraic equations, is compiled;
5) The solution of a system of the equations by the Gauss's method, definition of nodal displacements in the accepted design scheme;
6) correction of nodal displacements of a stressed armature owing to armature pre-stressing;
7) Definition of components of the intense deformed concrete and armature taking into account elastic work of materials before emersion of cracks.

All aforesaid is accepted in the capacity of initial material for the description of the concept of a fractal with the reference to the intense-deformed condition of a concrete slab.

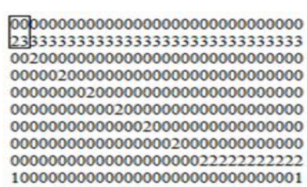
2. THE APPLICATION OF FRACTAL ANALYSIS

We use a formalism of the geometrical description of a multifractal. Let us divide consistently a slab on i blocks with the length $l_i \to 0$ (Aryassov *et al.*, 2010; Aryassov *et al.*, 2011). It is the general scheme of the geometrical description of any fractal array. Such geometrical way is reduced to sequence of $n \to \infty$ steps of division of the initial array, resulting in formation of N_n fragments of specific length $l_i \to 0, i = 1,2,\ldots,N_n$. The concrete slab reinforced in a certain way, will be presented as a random fractal array. Values of probabilities of realization of each fragment allow to define the function forming a fractal which in turn allows to estimate probability (P) of occurrence of a tension in a certain place of a slab, and also to estimate character of a state of stress (linear, flat, general state of stress). The probability of realization of each fragment is defined by the formula:

$$P_1 = l_1^{\alpha}, P_2 = l_2^{\alpha}, \ldots, P_{N_n} = l_{N_n}^{\alpha} \tag{3}$$

Let us combine a geometrical fragment with a matrix of numbers which has been made on the basis of characteristics of binding elements. Let us admit, at the smallest division the concrete matrix of numbers (fig. 1) was obtained. Then simple summation of numbers defines the general weight coefficient, for example:

$$\mu = 2 + 18 \cdot 2 + 30 \cdot 3 = 128 \tag{4}$$



Fiq-1. Matrix of numbers
Source: Allinson and Lawson (1990).

At the subsequent divisions we enlarge cages and again we make a matrix of numbers, for example on 4 cages, the square side doubles $l_2 = 2 \cdot l_1$. Some boundary sections appear discarded. Then probabilities of realization of the loaded fragments, probabilities of that the binding element or certain quantity of binding elements got to corresponding cages, allow to define the function forming a fractal.

At the second division it was gained, for example:

$$P_1 = \frac{5}{128}, P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = P_{10} =$$
$$= P_{11} = P_{12} = P_{13} = P_{14} = P_{15} = \frac{6}{128}; P_{16} = P_{17} = 0,$$
$$P_{18} = P_{19} = P_{37} = P_{38} = P_{56} = P_{57} = \frac{2}{128}, P_{20} \div P_{36} = 0, \quad (5)$$
$$P_{39} \div P_{55} = 0, P_{58} \div P_{64} = 0, P_{65} = \frac{1}{128}, P \div P_{74} = 0,$$
$$P_{75} \div P_{79} = \frac{4}{128}, P_{80} \div P_{96} = 0$$

At the third division other row of probabilities will be gained:

$$P_i = l_i^{\alpha}$$

Where

α–random quantity, $f(\alpha)$ -function of its distribution

Productive function (Ussanee and Voratas, 2003; Moonsoo *et al.*, 2008; Li-Chin and Sian-Kun, 2009; Alper *et al.*, 2014) of fractal measure which characterizes the loaded condition of a concrete slab:

$$M(q) = \sum_{i=1}^{N_n} P_i^q = \sum_{i=1}^{N_n} l_i^{\alpha q} \quad (6)$$

At N_1

$$M(q) = \sum_{i=1}^{N_1} P_i^q = \sum_{i=1}^{N_1} l_i^{\alpha q} \quad (7)$$

Where

i–current parameter along all divisions, α–scaling parameter, q–dimension of a fractal by means of which the intense-deformed condition of an object is assessed. From all array of the fragments, any the most feasible, which characterizes the actual picture of a state of stress of a concrete slab.

Corresponding scaling parameter is realized for:

$$N_n(\alpha) = l_n^{-f(\alpha)}$$

The less it 1_n, the more is $N_n.(\alpha)$.

$f(\alpha)$ – a distribution function of a random αvariable.

N_n –quantity of fragments are of the same length. Function $f(\alpha)$ sets dimensions of a quantity of geometrical array, defines a spectrum of α values.

Course-of-value function of a measure for the given $N_n(\alpha)$

$$M_n(q) = \sum_{n=1}^{N_n} l_n^{q\alpha - f(\alpha)} \tag{8}$$

At the uniform distribution law of random quantity α the course-of-value function can be presented as an expression:

$$M_n(q) = \int_{\alpha_{min}}^{\alpha_{max}} l_n^{q\alpha - f(\alpha)} \rho(\alpha)\, d\alpha \tag{9}$$

$\rho(\alpha)$ –density of a distribution of fragments l_n on parametere α. The greatest contribution to an assessment of a course-of-value function $M_n(q)$ is given by those values of parameter α, at which the parameter in sub integral expression (8) will aspire to a minimum, as the length l_n is small. The given condition means that:

$$\left.\frac{df}{d\alpha}\right|_{\alpha=\alpha(q)} = q, \quad \left.\frac{d^2 f}{d\alpha^2}\right|_{\alpha=\alpha(q)} < 0 \tag{10}$$

Taking into account the aforesaid and with application of a method of the quickest descent, expression (8) is led to a kind

$$M_n(q) = l_n^{q\alpha(q) - f(\alpha(q))} \tag{11}$$

The probability of realization of each fragment P_n^q estimate a course-of-value function $M_n(q)$. Taking into account the aforesaid, it is possible on the found values $\alpha(q)$, $f(\alpha(q))$ to define character of the intense-deformed condition of an object. The least squares method allows obtaining following dependences

$$(1+\frac{1}{2q\cdot\alpha^2})[\sum_{n=1}^{N_n} P_n \cdot l_n^{\alpha(q)-\frac{f(\alpha(q))}{q}} \cdot \ln l_n - \sum_{n=1}^{N_n} l_n^{2\alpha(q)-\frac{2f(\alpha(q))}{q}}] = 0$$

$$\frac{\alpha(q)q + f(\alpha(q))}{q\alpha} \times$$

$$\times\left[\sum_{n=1}^{N_n} P_n l_n^{\frac{\alpha(q)q - f(\alpha(q))}{q}} \ln l_n - \sum_{n=1}^{N_n} l_n^{\frac{2\alpha(q)q - 2f(\alpha(q))}{q}} \ln l_n\right] = 0$$

$$\left((1+\frac{1}{2\alpha^2 q})\cdot\frac{d\alpha(q)}{dq} + \frac{f(\alpha(q))}{q^2}\right)\times$$

$$\times\left[\sum_{n=1}^{N_n} P_n l_n^{\frac{\alpha(q)q - f(\alpha(q))}{q}} \ln l_n - \sum_{n=1}^{N_n} l_n^{\frac{2\alpha(q)q - 2f(\alpha(q))}{q}} \ln l_n\right] = 0$$

The gained magnitude of a measure $M(q)$ allows defining a spectrum of dimensions of a quantity of a multifractal which assesses a character of a state of stress of an object.

3. CONCLUSION

In the given paper the probabilistic nature is used only and geometrical features of a multifractal are not used. Taking into account the fact that the length l_n of a finite element aspires to zero, it is possible to make fragment numbering different, considering, that the probability of a damage of a whole object equals to one. Dependence of a course-of-value on dimension of a multifractal allows estimating the character of intense-deformed condition

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