The Study of Addition and Subtraction for Two Digit Numbers in Grade One Between Singapore and Taiwan

Der-Ching Yang[1]◎
Ming-Chen Chang[2]
Iwan Andi Sianturi[3]

[1,2,3]*National Chiayi University, Taiwan*

(◎ Corresponding Author)

ABSTRACT

This study applied the content analysis method to compare the differences of addition and subtraction for two digit numbers in grade one textbook. The published mathematics textbook of "Nani" in Taiwan and "My Pals Are Here [MPAH]" in Singapore were selected to be compared. They are the most popular elementary mathematics textbooks in each country. The results show that MPAH uses different approaches, including counting on, place value chart and standard written algorithm, to lead students to solve two digit plus one digit questions. The Nani textbook highlights the use of pictorial representation and horizontal algorithm. The MPAH textbook also teach two digits plus two digits after the two digits plus one digit in one chapter. However, The Nani textbook only focuses on two digits plus one digit and the two digits plus two digits will be taught at grade two. In addition, the Nani textbook emphasizes the introduction of special terms, such as summand, addend, sum, minuend, subtrahend, and difference. Yet the MPAH does not introduce these mathematical terms.

Keywords: *Addition, Grade one, Singapore, Subtraction, Taiwan.*

1. INTRODUCTION

1.1. Introduce the Problem

Cross-national comparison on mathematics textbooks has been considered as an essential issue to understand children's learning opportunities on mathematics (Haggarty and Pepin, 2002; Fan, 2013; Yang and Lin, 2015). The understanding of basic knowledge on addition and subtraction of whole numbers has been emphasized by earlier studies (Fuson, 1992; Ellemor-Collins and Wright, 2009). The development of facile thinking and use of multiple strategies to solve addition and subtraction problems also play a key role in daily life situations (Fuson, 1992; Yang and Huang, 2014). In addition, the design of questions presented in the mathematics textbooks will affect teachers' teaching and students' learning effectiveness (Tornroos, 2005; Xin, 2007; Fan, 2013). This shows the importance of this study.

Singapore and Taiwan students performed relatively at the top on International mathematics assessment, such as PISA and TIMSS (International Association for the Evaluation of Educational Achievement [IEA), 2016; Organisation for Economic Co-operation and Development (OECD), 2016). The two countries were selected because each has a national curriculum, and thus approved textbooks are likely to reflect the curriculum that public school students in each country are required to study. Moreover, the two countries differ in history, size, language, economy, culture, and student attainment in international comparative studies. We anticipated that these differences would be reflected in how opportunities to learn are expressed in the mathematics textbooks.

In addition, the science and mathematics education in Singapore has been considered as the best around the world (Ahuja, 2005; Hoven and Garelick, 2007). Many different reasons result in this excellent performance. Singapore has good mathematics textbook, is probably one of the reasons. Therefore, this study selected the Singaporean mathematics textbooks as a sample and tried to examine the differences on the topic of whole number addition and subtraction between Singapore and Taiwan. The research question is what are the differences on the designs of whole number addition and subtraction between Singapore and Taiwan?

1.2. Mathematics Textbooks Related Studies

Cross-national textbooks study became an important issue due to its role in teaching and learning, especially in international comparison (Fan, 2013). Previous studies analyzed textbooks from different countries to find the advantages and disadvantages of textbooks, so the results could shed light to designing future textbooks (Charalambous *et al.*, 2010; Son, 2012; Fan, 2013).

Earlier studies have reported that mathematics textbooks play an important role in teaching and learning of mathematics (Schmidt *et al.*, 2001; Weinberg and Weisner, 2010; Fan, 2013). Textbooks also are an independent tool to help students to learn mathematics (Zhu and Fan, 2006; Sood and Jitendra, 2007). It is well documented that the quality of textbook contents will affect students' learning and, in this case, it will directly impact their performance (Haggarty and Pepin, 2002; Zhu and Fan, 2006). Moreover, the representation methods of mathematics, problem types, and the order and sequence of contents will influence students' learning effect (Haggarty and Pepin, 2002; Zhu and Fan, 2006; Sood and Jitendra, 2007). Based on these arguments, this study will analyze the differences of on the designs of whole number addition and subtraction between Singapore and Taiwan.

In terms of international textbook study, Charalambous *et al.* (2010) found that Taiwanese textbook provided more questions requiring higher cognitive demand level than Cyprus and Ireland textbooks in fraction units, and it also provided more questions which need the answer, mathematical sentence, and explanation. On the other hand, Erbas *et al.* (2012) reported that Singapore textbooks reflected simple features of text density and enriched use of visual elements, fewer number of topics, and an easier inner organization to follow. Moreover, Singaporean

textbooks are used in some school districts in the United States as teachers and mathematicians like their simple approach (Hoven and Garelick, 2007).

1.3. Whole Number Addition and Subtraction Related Studies

Children's learning of addition and subtraction was the subject of considerable research in the last two decades (e.g., (Kilpatrick *et al.*, 2001; Verschaffel *et al.*, 2007; Barrouillet *et al.*, 2008)). Concerning the contents of textbooks, each country has its own design to teaching addition and subtraction in the early grades. How teachers can help children master the basic addition and subtraction is an important, long-standing issue in every country in the world. Educators in different countries have developed unique approaches to teaching basic addition and subtraction (e.g., (Sun and Zhang, 2001); Kessel and Ma (2011)). In very early learning of addition and subtraction, children use strategies involving counting by ones, and will rely on visible objects to count. Later, children can count visualized objects, fingers, and their own recited counting words ((Steffe and Cobb, 1988).

National Council of Teachers of Mathematics [NCTM] (2000) suggests that fluency with basic addition and subtraction number combinations is a goal in teaching whole-number computation. A mastery of lower-order skills instills confidence in students and facilities higher-order thinking. Some problems which involve using one addition or subtraction operation can be difficult for students until they are between ages 12 and 14 (Vergnaud, 2009). This difficulty often relates to the nature of problems as mathematical constructs. In order to better understand possible sources of students' difficulties in solving addition and subtraction problems, it is crucial to first look at the mathematical nature of this type of task. Therefore, we concerned on what are the differences on the designs of whole number addition and subtraction between Singapore and Taiwan.

2. METHOD

The content analysis method was used to examine the differences on the designs of whole number addition and subtraction between Singapore and Taiwan.

2.1. The Sample

In this study, we selected **Nani mathematics textbooks** from Taiwan and **MY PALS ARE HERE**"! Maths {MPHM}" from Singapore. Currently, there are three (3) different elementary mathematics textbook series used in Taiwan. Nani textbook is the most popular textbook used in Taiwan and the percentage of its market share is about 44% (Nani Publishing Company, 2016). On the other hand, MY PALS ARE HERE is the most popular elementary mathematics textbooks used in Singapore and the percentage of market share is about 70% (Yang *et al.*, 2010). The units which focus on addition and subtraction for two digit whole numbers in grade one were selected.

2.2. Data Coding and Analysis

This study only analyzed the students' textbooks. A question, including worked examples and exercises, presented in the student textbook which was considered as the smallest analysis unit. For example, in the following Figure 1, 24 + 3 = ?, was counted as one question.
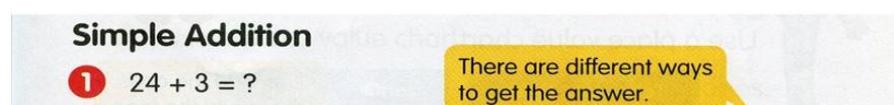
Figure-1. An example from MPHM
Note: Adapted from MPHM 1B, p. 37.

2.3. Analytical Framework

The types of whole number addition and subtraction were divided into two subcategories: two digits whole numbers add/subtract one digit whole number (e.g., $27 - 6 = ?$) and two digits add/subtract two digits whole numbers (e. g., $15 + 12 = ?$). The presentation styles of whole number addition and subtraction were divided into two subcategories: Purely mathematical problem (e. g., $24 + 3 = ?$) and application problem (e. g., John has $29 dollars. He spent $5 dollars to buy a pen. How much money did he have after he bought the pen?).

2.4. Inter-Reliability

To ensure reliability, two coders (authors) individually coded all of the questions in the two textbook series based on the analytical framework. In addition, to examine the overall accuracy of the coding, the inter-reliability of coding was checked between the coders. The final reliability was 0.92. This indicated that this study has good inter-reliability.

3. RESULTS

Table 1 reports the frequencies of types of whole number addition and subtraction. The results show that grade one math textbooks in Singapore include two digits add/subtract one digit numbers and two digits add/subtract two digits numbers. However, grade one textbooks in Taiwan only teach two digits add/subtract one digit numbers. The two digits add/subtract two digits numbers will be taught in grade two in Taiwanese textbooks.

Table-1. The frequencies of types of whole number addition and subtraction

	Two digits add/subtract one digit	Two digits add/subtract two digits	Total
MPHM	16 (41%)	23 (59%)	39
Nani	53 (100%)	0 (0%)	53

Note: Data adapted from MPHM and Nani and created by authors

Table 2 reports the frequencies of presentation styles of whole number addition and subtraction. The textbooks in Singapore put more emphasis on purely mathematical problems (over 3/4) than the Taiwanese textbooks (less than 1/3).

Table-2. The frequencies of styles of whole number addition and subtraction

	Purely mathematical problem	Application problem	Total
MPHM	30 (76.9%)	9 (23.1%)	39
Nani	33 (62.2%)	20 (37.8%)	53

**Note:** Data adapted from MPHM and Nani and created by authors

The data show that the MPHM uses different approaches, including counting on, place value chart and standard written algorithm, to lead students to solve two digit plus one digit questions. For example, Figure 2 shows the different ways to get the answer which used in MPHM.
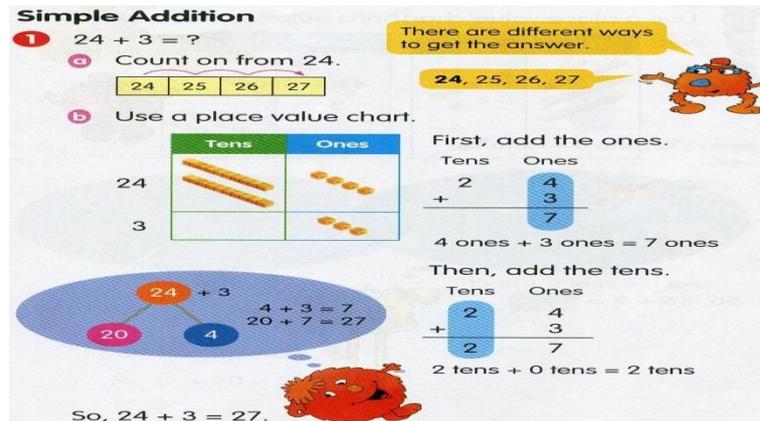
Figure-2. An example from MPHM
Note: Adapted from MPHM 1B, p. 37

However, the Nani textbook I Taiwan highlights the use of pictorial representation and horizontal algorithm to teach children learn how to solve problems. Figure 2 presents the example from Taiwan Nani.
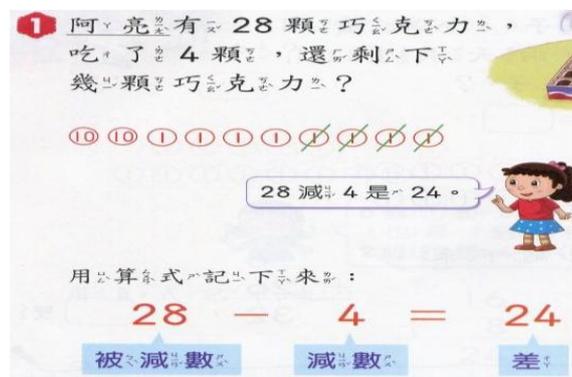
Figure-3. An example from Nani textbook
Note: Adapted from Nani textbook, 1B, p. 92.

In addition, the Nani textbook emphasizes the introduction of special terms, such as summand, addend, sum, minuend, subtrahend, and difference (see Figure 3). Yet the MPAH does not introduce these mathematical terms.

Figure-4. Two examples from Nani textbook
Note: Adapted from Nani textbook, 1B, p. 88, 92.

4. DISCUSSION AND CONCLUSION

The findings show that the MPHM for grade one in Singapore includes more depth contents than the Nani in Taiwan. Grade-one math textbooks in Singapore already teach two digits to add/subtract two digits numbers. However, the same topic two digits add/subtract two digits numbers in Nani of Taiwan is taught in grade two and in grade-one textbooks only teach two digits add/subtract one digit numbers. It seems reasonable to believe that the Singapore textbooks include more difficult contents at the beginning of the elementary school mathematics. The finding is similar to the earlier studies that Singapore textbooks treated mathematics contents more difficult and

included in the earlier grade than the other countries (Yang *et al.*, 2010). The results also show that the difference on the presentation styles of mathematics problems. There are fewer questions that two-thirds of questions in Taiwan highlighted the purely mathematics form and over one-thirds of questions belong to application form. On the other hand, there are about three-fourths of questions in Singapore put more emphasis on purely mathematics form and fewer questions that one-fourths of questions dealt with application form.

The findings also showed that MPHM used different approaches, including counting on, place value chart, and standard written algorithm, to lead students to solve two digit add/subtract one or two digits questions. However, the Nani textbook highlighted the use of pictorial representation and horizontal algorithm. Earlier studies suggested that the use of multiple representations will help children's conceptual understanding (Zhu and Fan, 2006; Yang and Huang, 2014). We do believe that students in Singapore perform well on international mathematics assessment. One of the key factors is probably due to using multiple representations to help children develop conceptual understanding at the earlier grade level.

Result also showed that the Nani textbook emphasizes the introduction of special terms, such as summand, addend, sum, minuend, subtrahend, and difference. Yet the MPAH does not introduce theses mathematical terms. Earlier studies indicated that mathematics textbooks may "vary greatly across countries in their development as well as the final product" (Yang *et al.*, 2010). The finding of this study is consistent with the earlier studies that different countries may treats textbooks in different ways (Schmidt, 2004; Yang *et al.*, 2010; Fan, 2013).

Finally, this study only focused on addition and subtraction for two digit numbers in grade one textbook. The generalization of the findings from this study should be careful. More related studies should be conducted to inform the strengths and weaknesses of different countries' mathematics textbooks. This will help policy makers and textbooks designers to revise theie countries' textbooks.

REFERENCES

Ahuja, O.P., 2005. World-class high quality mathematics education for all K-12 American students. Montana Mathematics Enthusiast, 3(2): 223-248. *View at Google Scholar*

Barrouillet, P., M. Mignon and C. Thevenot, 2008. Strategies in subtraction problem solving in children. Journal of Experimental Child Psychology, 99(4): 233–251. *View at Google Scholar | View at Publisher*

Charalambous, C.Y., S. Delaney, H.Y. Hsu and V. Mesa, 2010. A comparative analysis of the addition and subtraction of fraction in textbooks from three countries. Mathematical Thinking and Learning, 12(2): 117-151. *View at Google Scholar | View at Publisher*

Ellemor-Collins, D. and R.J. Wright, 2009. Structuring number 1 to 20: Developing facile addition and subtraction. Mathematics Education Research Journal, 21(2): 50-75. *View at Google Scholar | View at Publisher*

Erbas, A.K., C. Alacaci and M. Bulut, 2012. A comparison of mathematics textbooks from Turkey, Singapore, and the United States of America. Educational Sciences: Theory & Practice, 12(3): 2324-2330. *View at Google Scholar*

Fan, L., 2013. Textbook research as scientific research: Towards a common ground on issues and methods of research on mathematics textbooks. ZDM Mathematics Education, 45(5): 765-777. *View at Google Scholar | View at Publisher*

Fuson, K.C., 1992. Research on whole number addition and subtraction. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning. New York: Macmllian. pp: 243-275.

Haggarty, L. and B. Pepin, 2002. An investigation of mathematics textbooks and their use in English, French, and German classrooms: Who get an opportunity to learn what? British Educational Research Journal, 28(4): 567–590. *View at Google Scholar | View at Publisher*

Hoven, J. and B. Garelick, 2007. Singapore math: Simple or complex? Educational Leadership, 65(3): 28-31. *View at Google Scholar |*

International Association for the Evaluation of Educational Achievement [IEA), 2016. TIMSS 2015 and TIMSS advanced 2015 international results. Boston, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.

Kessel, C. and L. Ma, 2011. Three approaches to one-place addition and subtraction: Counting strategies, memorized facts, and thinking tools. Retrieved from http://lipingma.net/math/One-place-number-addition-and-subtraction-Ma-Draft-2011.pdf [Accessed February 23, 2017].

Kilpatrick, J., J. Swafford and B. Findell, 2001. Adding it up. Helping children learn mathematics. Washington, DC: National Academy Press.

Nani Publishing Company, 2016. Nani elementary mathematics textbooks. Tainan: Nani Publishing Company.

National Council of Teachers of Mathematics [NCTM], 2000. Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

Organisation for Economic Co-operation and Development (OECD), 2016. PISA 2015 results in focus. Retrieved from https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf [Accessed December 20, 2016].

Schmidt, W.H., 2004. A vision for mathematics. Educational Leadership, 61(5): 6-11. *View at Google Scholar*

Schmidt, W.H., C.C. McKnight, R.T. Houang, H. Wang, D.E. Wiley, L.S. Cogan and R.G. Wolfe, 2001. Why schools matter: A cross-national investigation of curriculum and learning. San Fransisco, CA: Jossie-Bass.

Son, J., 2012. A cross-national comparison of reform curricula in Korea and the US in terms of cognitive complexity: The case of fraction addition and subtraction, ZDM. International Journal of Mathematics Education, 44(2): 161-174. *View at Google Scholar | View at Publisher*

Sood, S. and A.K. Jitendra, 2007. A comparative analysis of number sense instruction in reform- based and traditional mathematics textbooks. Journal of Special Education, 41(3): 145-157. *View at Google Scholar | View at Publisher*

Steffe, L.P. and P. Cobb, 1988. Construction of arithmetic meanings and strategies. New York: Springer.

Sun, W. and Y.Z. Zhang, 2001. Teaching addition and subtraction facts: A Chinese perspective. Retrieved from http://www.nctm.org/Publications/teaching-children-mathematics/ [Accessed February 22, 2016].

Tornroos, J., 2005. Mathematics textbooks, opportunity to learn and student achievement. Studies in Educational Evaluation, 31(4): 315–327. *View at Google Scholar | View at Publisher*

Vergnaud, G., 2009. The theory of conceptual fields. Human Development, 52(2): 83–94. *View at Google Scholar | View at Publisher*

Verschaffel, L., B. Greer and E. De Corte, 2007. Whole number concepts and operations. In F. K. Lester (Ed.), Second handbook of research in mathematics teaching and learning. New York: Macmillan. pp: 557–628.

Weinberg, A. and E. Weisner, 2010. Understanding mathematics textbooks through reader-oriented theory. Educational Study in Mathematics, 76(1): 49-63. *View at Google Scholar | View at Publisher*

Xin, Y.P., 2007. Word problem solving tasks in textbooks and their relation to student performance. Journal of Educational Research, 100(6): 347–359. *View at Google Scholar | View at Publisher*

Yang, D.C. and K.L. Huang, 2014. An intervention study on mental computation for 2nd graders in Taiwan. Journal of Educational Research, 107(1): 3-15. *View at Google Scholar | View at Publisher*

Yang, D.C. and Y.C. Lin, 2015. Examining the differences of linear systems between Finnish and Taiwanese textbooks. Eurasia Journal of Mathematics, Science & Technology Education, 11(6): 1-17. *View at Google Scholar*

Yang, D.C., R.E. Reys and L.L. Wu, 2010. Comparing how fractions were developed in textbooks used by the 5th- and 6th-graders in Singapore, Taiwan, and the U.S.A. School Science and Mathematics, 110(3): 118-127. *View at Google Scholar | View at Publisher*

Zhu, Y. and L. Fan, 2006. Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and the United States. International Journal of Science and Mathematics Education, 4(4): 609-626. *View at Google Scholar | View at Publisher*