How Did Mathematics Postgraduates Obtain Tacit Knowledge of Mathematical Problem Solving?

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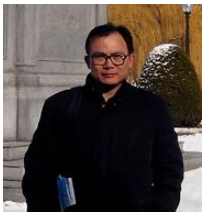

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ABSTRACT

This research focused on the obtainment of mathematics postgraduates' tacit knowledge of Mathematics problem solving, investigated 172 mathematics postgraduates with a closed-ended structured questionnaire which was formed based on the early interview. The results indicated that mathematics postgraduates' tacit knowledge of Mathematics problem solving mainly came from their teachers' explanation and demonstration about mathematics problem solving, their practice of mathematics problem solving, and their learning to knowledge of mathematics problem solving from related book. The other factors of teaching and learning impacted on the obtainment of tacit knowledge, but their contribution rate was not large. So we suggest that the college mathematics teachers should teach mathematics problem solving with Apprentice style and pay attention to guiding students to learn knowledge about mathematics problem solving.

Keywords: *Tacit knowledge, Mathematics problems, Explanation, Demonstration, Postgraduates.*

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1. INTRODUCTION

College mathematics teaching aims to help students mastering the modern content of mathematics, develop their mathematical thinking, improve their ability of analyzing and solving problems with relevant mathematics knowledge and lay the foundation for them to move towards society and engage in work about technology and economy. To improve students' problem solving ability, teachers must pay attention to the teaching of tacit knowledge and make them master more relevant tacit knowledge. Tacit knowledge refers to the knowledge, in mathematics problem solving process, used to generalize the conditions, analyze the problems, select the methods, and properly invoke mathematics knowledge that has been studied before to rationally solve the new problems. This part of knowledge plays an important role in the process of solving mathematics problems. It is the basis and guarantee for individuals to solve mathematics problems smoothly (Lv, 2014). But how can students get this part of knowledge effectively? No definite or unified answer appeared yet. Polanyi (1993) pointed out that this part of knowledge were non-logical and non-public. At present, many researchers believed that the best way was to make this part of knowledge explicit and definite, so we should further strengthen the teaching of this part of knowledge (Shi, 2001; Huang, 2005; Jiang, 2010; Lu, 2010; Yi, 2013; Li, 2015; Sun, 2015; Xu, 2015; Yu, 2015; Gao, 2016; Wang and Yu, 2016; Zhou, 2017). However, some scholars believed that the best way was to let the students experience and comprehend, so to strengthen students' learning was necessary (La, 2016). What should we do exactly? We think that conducting an investigation to the students who have high mathematics problem solving ability to find out their sources of tacit knowledge about Mathematical problem solving is undoubtedly valuable and meaningful. For this reason, we had investigated senior students in mathematics in 2013. The results we obtained indicated their tacit knowledge of Mathematical problem solving mainly came from their teachers' explanation and demonstration about mathematics problem solving, their practice of mathematics problem solving, and their learning to the knowledge of mathematics problem solving from related book (Yang, 2014). Is it the same for postgraduates in mathematics? We chose 172 postgraduates in mathematics of five provincial universities in Shandong Province and conducted an investigation.

2. METHODOLOGY

2.1. Participants

We chose 172 postgraduates in mathematics as participants from five provincial universities in Shandong Province, among which included 87 fresh postgraduates,75 second-year postgraduates , 51 male postgraduates and 121 female postgraduates, 97 postgraduates are in Basic Mathematics, 42 postgraduates are in Applied Mathematics, and 33 postgraduates are in Computational Mathematics.

2.2. Instrument

We adopted self-designed questionnaire on the base of early interview, which was conducted among postgraduates in mathematics. The questions of early interview was that "How do you get the knowledge about mathematics problem solving?". According to the results of the early interview, we divided the tacit knowledge about Mathematical problem solving into seven categories and thirteen small classes. The seven categories are: (1) the knowledge of understanding mathematical problems, (2)the knowledge of mastering the key of mathematics problems, (3)the knowledge of linking mathematics problems to old knowledge, (4)the knowledge of finding solutions, (5)the knowledge of self monitoring in problem solving process, (6)the knowledge of rethinking problem solving and (7)the knowledge of applying mathematics theory to solve practical problems. The thirteen small classes is the subdivision of the seven categories mentioned above.

The questionnaire sets up problems for each small class of tacit knowledge, and gives six fixed answers and an open answer to each question. The options are: A. Obtained from teacher's explanation, B. Obtained from teacher's problem solving demonstration, C. Obtained from explanation of classmates or friends, D. Obtained from classmates' problem solving demonstration, E. Obtained from related books (text, graphics, examples, etc.) about knowledge of mathematics problem solving, F. Obtained from practice of mathematics problem solving, and G. Other, please write down the details under the corresponding questions. Both multiple choice and individual choice were possible. In reply, postgraduates in mathematics were asked not only to fill out the answer, but also to write down the contribution rate (percentage) of the options for acquiring tacit knowledge---that is, to write down the contribution rate of teacher's explanation, teacher's demonstration, classmates' explanation, classmates' demonstration, their learning and their practice for acquiring tacit knowledge of mathematics problem solving respectively. The questionnaire was revised several times according to the expert's advice, and finally achieved higher validity. The test-retest reliability of the questionnaire was 0.91.

2.3. Data Collection

The questionnaire was sent and recalled by the examiner and the teacher of the postgraduates. We recalled 172 questionnaires after they were sent one day later. After eliminating the ineffective questionnaires, we obtained 169 effective questionnaires finally, and the effective rate was 98.26%. The three ineffective questionnaires were mainly due to its answers incomplete.

3. RESULTS

3.1. The Obtainment of Tacit Knowledge about Understanding Mathematics Problems

The tacit knowledge about mathematics problems understanding refers to that knowledge about how to understand the text, graphics, symbols and formulas of mathematics problems. According to the response of postgraduates, there were 82.96% postgraduates choosing answer B, 82.61% postgraduates choosing answer A, 78.26% postgraduates choosing answer E, 73.91% postgraduates choosing answer F, 38.23% postgraduates choosing answer C and 42.89% postgraduates choosing answer D. As for the contribution rate, the average of the above six aspects mentioned above are 28.22, 22.65, 5.27, 4.31, 20.44 and 19.13. The response of all postgraduates is as shown in Figure 1.

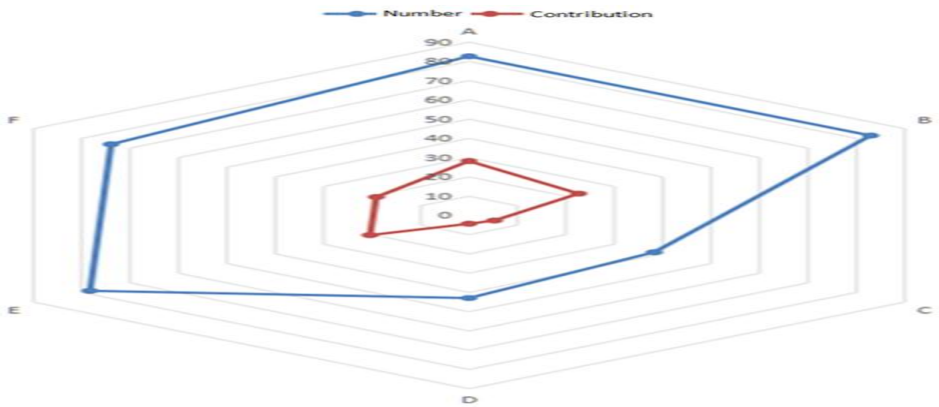

Fig-1. Response about tacit knowledge No.1.
Source: Researchers' analysis of answered questionnaires , June, 2016

As illustrated in Figure 1, it is obvious that most postgraduates' tacit knowledge of understanding mathematics problems are mainly from the teacher's explanation, the teacher's demonstration about problem solving and their

learning to knowledge of mathematical problem solving from related book. Not only that, the contribution rates of the teacher's explanation, the teacher's demonstration and their learning for obtaining tacit knowledge about understanding mathematics problems are also large. Although about half of the postgraduates thought that the explanation and demonstration of classmates and friends were the source of their tacit knowledge in understanding mathematics problems, it could be seen from Figure 1 that their contribution rate was not large.

3.2. The Obtainment of Tacit Knowledge About Mastering the Key of Mathematics Problems

The tacit knowledge about mastering the key of mathematics problems refers to that about summing up the contents of mathematics problems, clarifying the conditions and key points of mathematics problems after understanding the characters, figures and symbols of problems. This part of knowledge can be further divided into two categories: One is to generalize the knowledge of problems (Knowledge I), and the other is to find out the key points of knowledge (Knowledge II). Most of the postgraduates chose the answers A, B, E and F. The postgraduates' choices are shown in Table 1.

Table-1. Postgraduates' choices about knowledge No.2

Percentage	Knowledge I		Knowledge Ⅱ	
	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)
A	86.96	27.14	78.26	19.57
B	84.31	24.78	91.33	28.91
C	12.78	2.98	10.32	5.43
D	14.52	3.22	19.87	9.35
E	82.61	16.13	65.22	14.35
F	87.26	25.17	78.26	22.39

Source: Researchers' analysis of answered questionnaires , June, 2016

The response of all postgraduates is as shown in figure 2.

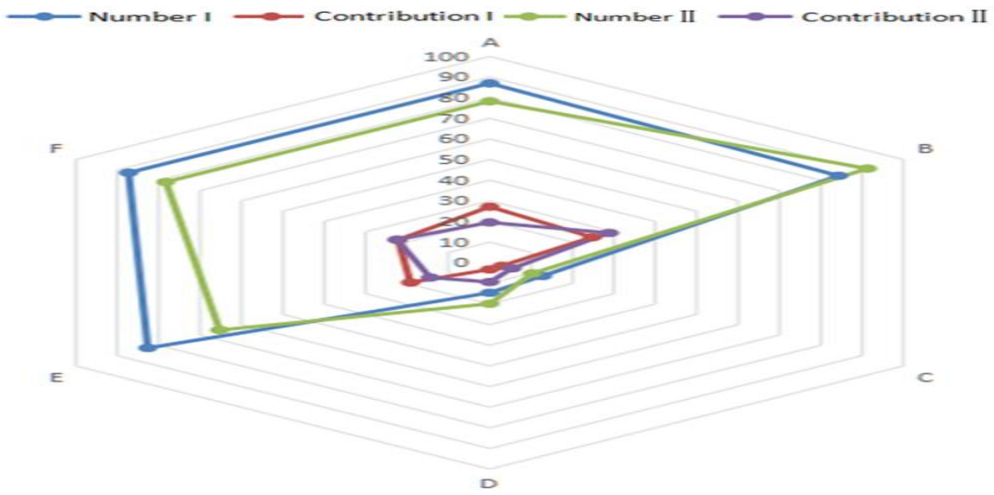

Fig-2. Response about tacit knowledge No.2
Source: Researchers' analysis of answered questionnaires , June, 2016

It can be seen from table 1 and Figure 2, most postgraduates believed that knowledge I came mainly from the teacher's explanation, the teacher's demonstration about mathematical problem solving, their practice of mathematics problem solving and their learning to knowledge of mathematical problem solving from related book.

Not only that, the contribution rate of the above four aspects is also large. For the knowledge II, most postgraduates believed that it mainly came from the teacher's demonstration about mathematics problem solving, the teacher's explanation and their learning. The large contribution to the formation of the knowledge II are the teacher's demonstration about mathematics problem solving and their practice of mathematics problem solving. For the helping of students or friends, as can be seen from Figure 2, not only few postgraduates chose it, and its contribution rate was also very low.

3.3. The Obtainment of Tacit Knowledge About Linking Mathematics Problems to Old Knowledge

This part of knowledge can be further divided into three categories. The first kind of knowledge is the knowledge about finding the connection between mathematics problems and old knowledge (Knowledge I). The second is the knowledge about turning a new problem into an old one (Knowledge II). The third kind is the knowledge about characterizing new problems with old knowledge properly (Knowledge III). In response to these three categories of knowledge, most postgraduates chose answers A, E and F. The postgraduates' specific choices are shown in table 2.

Table-2. Postgraduates' choices about knowledge No.3

Percentage	Knowledge Ⅰ		Knowledge Ⅱ		Knowledge Ⅲ	
	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)
A	91.30	28.48	86.96	31.96	95.65	31.09
B	73.91	21.09	73.91	19.13	73.91	16.09
C	52.17	8.70	47.83	7.39	47.83	7.39
D	43.48	5.87	43.48	5.22	34.78	3.91
E	73.91	22.39	82.61	20.22	86.96	19.13
F	60.87	13.48	65.22	16.09	86.96	22.39

Source: Researchers' analysis of answered questionnaires , June, 2016

The response of all postgraduates is as shown in figure 3.

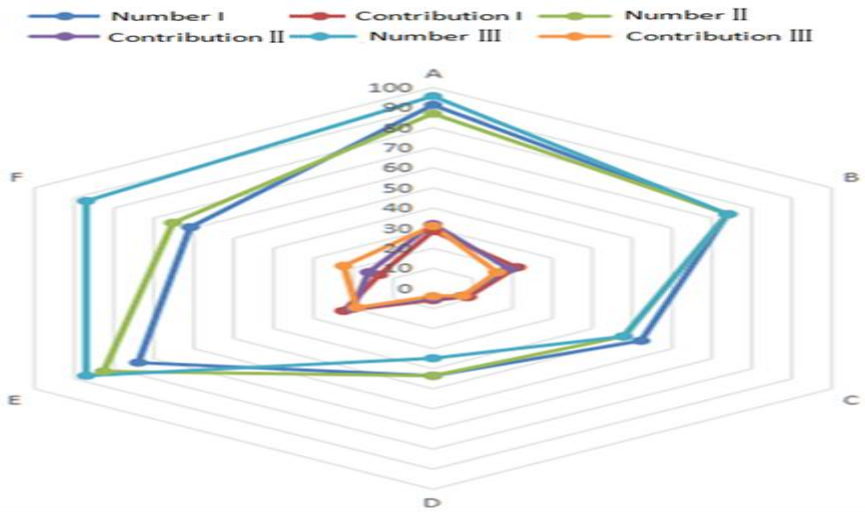

Fig-3. Response about tacit knowledge No.3
Source: Researchers' analysis of answered questionnaires , June, 2016

From table 2 and figure 3 it can be seen that, for the above three kinds of knowledge, the vast majority of the postgraduates thought it mainly came from the teacher's explanation, followed by their learning to knowledge of mathematics problem solving from related book, their practice of mathematics problem solving, and teacher's demonstration. For the obtainment of the knowledge I, the large contribution rates are in the teacher's explanation, their learning about knowledge of mathematics problem solving from related book, their practice of mathematics problem solving and the teacher's demonstration. For the knowledge II and III, the large contribution are in the teacher's explanation, their own learning and the teacher's demonstration. For the helping of classmates and friends, although almost 40% postgraduates thought that they had played a role in the obtainment of this tacit knowledge, but the contribution rate was not very large.

3.4. The Obtainment of Tacit Knowledge About Finding Solutions

This part of knowledge also can be further divided into three parts, the first part is the tacit knowledge about selecting and extracting old knowledge for new problems (Knowledge I), the second part is the tacit knowledge about doing the auxiliary line and auxiliary function according to the characteristics and requirements of the problem (Knowledge II), the third part is the tacit knowledge about finding a fast and simple method (Knowledge III). Most postgraduates chose the answers A, B, E, and F. The postgraduates' specific choices are shown in table 3.

Table-3. Postgraduates' choices about knowledge No.4

Percentage	Knowledge I		Knowledge Ⅱ		KnowledgeⅢ	
	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)
A	82.61	24.35	86.96	21.30	91.30	27.83
B	82.61	22.83	100.00	31.96	78.26	19.13
C	43.48	4.13	47.83	5.22	69.57	11.09
D	56.52	8.26	69.57	10.87	56.52	6.96
E	73.91	18.04	73.91	12.83	78.26	18.70
F	86.96	22.39	82.61	17.83	69.57	16.30

Source: Researchers' analysis of answered questionnaires , June, 2016

The response of all postgraduates is as shown in figure 4.

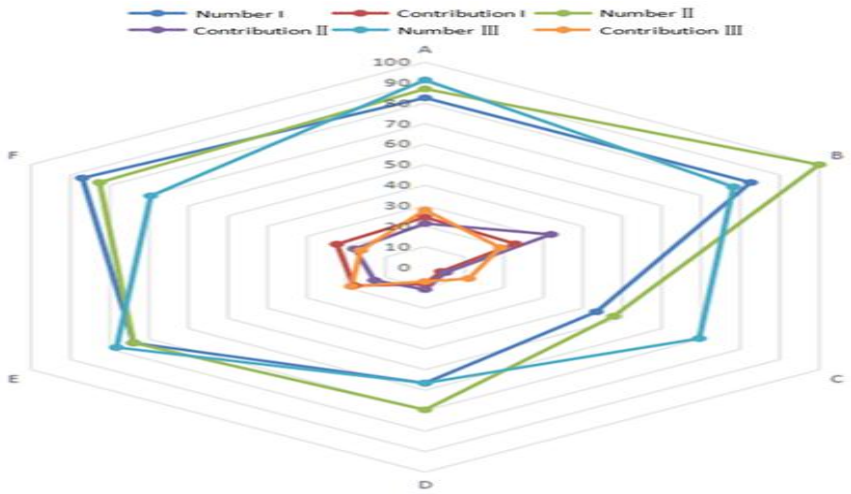

Fig-4. Response about tacit knowledge No.4
Source: Researchers' analysis of answered questionnaires , June, 2016

As illustrated in table 3 and Figure 4, for the obtainment of the knowledge I and II, most postgraduates thought that the main sources were the teacher's explanation, teacher's demonstration of problem solving, their learning to knowledge of mathematics problem solving from related book, their practice of mathematics problem solving and the explanation and demonstration of classmates and friends. Moreover, these five aspects are also the most important source, especially the teacher's demonstration of problem solving. For the obtainment of the knowledge III, most postgraduates believed that the above six aspects were the main and important sources, just the contribution of the helping of classmates and friends is small.

3.5. The Obtainment of Tacit Knowledge About Self Monitoring During the Process of Problem Solving

The tacit knowledge about self monitoring during the process of problem solving is the knowledge that the problem solver used to constantly adjust the direction and method and check the errors in the process of mathematics problem solving. It can be further divided into two parts---one is that to adjust the direction and method of solving problems constantly (Knowledge I), the other is that to correct the mistakes (Knowledge II). For the two parts of tacit knowledge, the postgraduates' specific choices are shown in Table 4.

Table-4. Postgraduates' choices about knowledge No.5

Percentage	Knowledge I		Knowledge Ⅱ	
	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)	Number of postgraduates (The total is 169)	Contribution Rate (The total is 100)
A	65.22	20.22	91.30	34.35
B	73.91	19.13	60.87	19.13
C	73.91	13.26	52.17	9.57
D	60.87	12.17	39.13	5.22
E	65.22	15.43	60.87	15.87
F	69.57	19.78	56.52	15.87

Source: Researchers' analysis of answered questionnaires , June, 2016

The response of all postgraduates is as shown in figure 5.

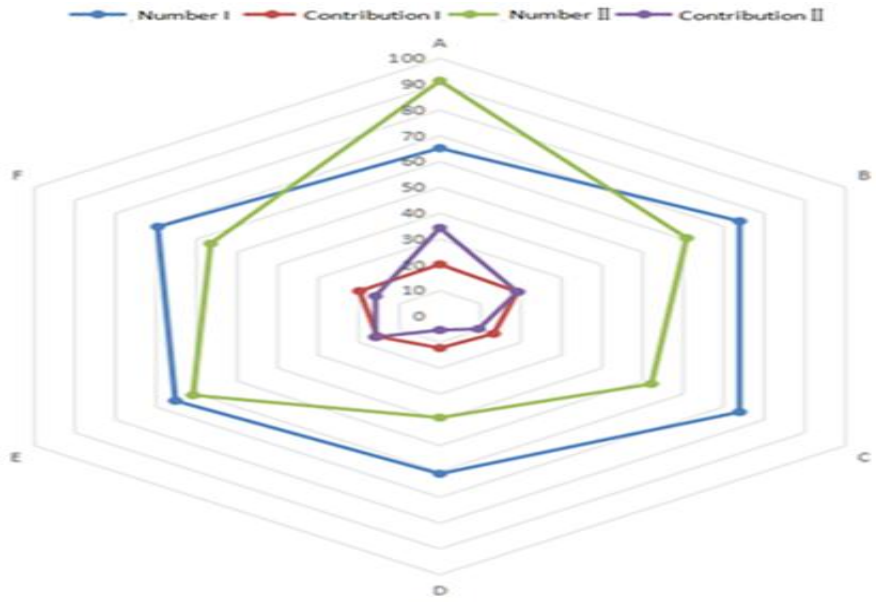

Fig-5. Response about tacit knowledge No.5
Source: Researchers' analysis of answered questionnaires , June, 2016

As can be seen in Table 4 and figure 5, more than half of postgraduates believed that the six aspects were the main and important sources, and each of them had a relatively average contribution rate. For the obtainment of the knowledge II, the vast majority of postgraduates thought that the teacher's explanation was the main source, and it also had the largest contribution rate.

3.6. The Obtainment of Tacit Knowledge about Rethinking Problem Solving

For the obtainment of these tacit knowledge, 78.26% postgraduates chose the answer F, 73.91% postgraduates chose the answer A, 69.57% postgraduates chose the answer E and 65.22% postgraduates chose the answer B. Moreover, according to the responses, the activities mentioned in these four options also have large contribution rate, especially the practice of postgraduates---its contribution rate is 33.48% of the total. Although not many postgraduates chose the other two answers, they were all around 50% yet. However its contribution rates were obviously low, only about 7%.

The response of all postgraduates is as shown in figure 6.

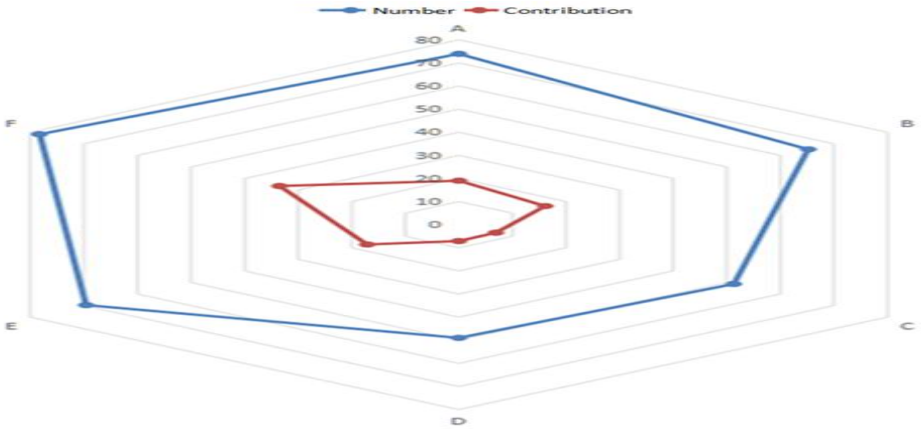

Fig-6. Response about tacit knowledge No.6
Source: Researchers' analysis of answered questionnaires , June, 2016

As you can see from the figure 6, the postgraduates' practice of mathematics problem solving contributes most to the obtainment of this tacit knowledge.

3.7. The Obtainment of Tacit Knowledge About Solving Practical Problems

To solve practical problems, we should understand not only the data and graphics, but also the relevant knowledge of the practice. Not only the general mathematical method but also some special methods should be considered. Therefore, in the process of solving practical problems, lots of tacit knowledge usually be used. For these kind of tacit knowledge, 91.30% postgraduates chose the answer F, 86.96% postgraduates chose the answer E, 82.61% postgraduates chose the answer A and 82.61% postgraduates chose the answer B. Moreover, according to the postgraduates' response, the practice of mathematics problem solving and the learning to mathematics problem solving from related books, and the teachers' explanation and demonstration about mathematics problem solving contributed more to their tacit knowledge, reaching 25%, 19.78%, 23.48% and 16.96% respectively. Although the number of the other two options' holders is about 60%, the contribution rate is obviously low---only about 7.4%.
The response of all postgraduates is as shown in figure 7.

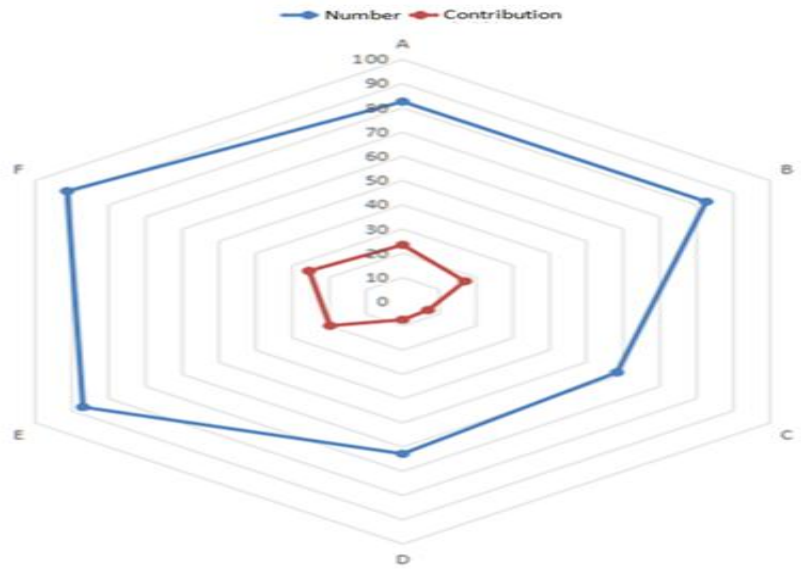

Fig-7. Response about tacit knowledge No.7.
Source: Researchers' analysis of answered questionnaires , June, 2016

As can be seen from Figure 7, most of the postgraduates thought that their practice of mathematics problem solving was the main and important source of their tacit knowledge.

4. CONCLUSIONS AND REFLECTIONS

Postgraduates in mathematics are proficient in solving mathematics problems among college students. The reasons for their relatively strong ability to solve mathematics problems should be that they not only have mastered mathematics knowledge but also have more tacit knowledge of mathematics problem solving. Understanding how they obtain tacit knowledge is obviously valuable and meaningful to guide the problem-solving study of junior students and unskilled students. Through the investigation to 172 postgraduates in mathematics , we find:

Firstly, for the postgraduates in mathematics, the obtainment of tacit knowledge in understanding mathematics problems and mastering the key of mathematics problems are mainly from the teacher's explanation, their learning about knowledge of mathematics problem solving from related book and practice of mathematics problem solving. The average contribution rate of these four activities in helping obtaining tacit knowledge is about 20%. The sum of all contribution rates is above 80%.

Secondly, for the postgraduates in mathematics, the obtainment of tacit knowledge about finding solutions, self monitoring in problem solving process, rethinking problem solving and applying mathematical theory to solve problem come from many sources. It includes not only four aspects mentioned above, but also the explanation and demonstration of classmates or friends. However, the contribution rate of the explanation and demonstration of classmates or friends is not too large for the obtainment of tacit knowledge.

From this point of view, the teachers' explanation and demonstration of mathematics problems, their learning to knowledge of mathematics problem solving from related book and practice of mathematics problem solving are the main and important sources of the tacit knowledge. This is almost the same as our previous research among senior students (Yang, 2014). Therefore, we should pay attention to the following aspects in order to improve students' mathematics problem solving ability as quickly and effectively as possible in the teaching of college mathematics.

Firstly, in the teaching of college mathematics, mathematics teachers should explain the analysis, solution and reflection process of mathematics problems solving detailedly to make students learn more tacit knowledge about

understanding mathematics problems, mastering the key of mathematics problems, reasonably seeking solutions to problems and properly using mathematics knowledge to solve the practical problems.

Secondly, in the process of solving the college mathematical problems, mathematics teachers should pay attention to showing their thinking process to students and expose their analysis and solving process to make students not only learn how to understand and use mathematics knowledge to solve problems, but also how to master the key point of the problems, how to analyze and explore the method to solve the problem, how to select the old knowledge to solve new problems and how to adopt reasonable methods for the characterization of problem solving.

Thirdly, in the process of problem solving, mathematics teachers should inspire students to study extensively and think deeply. Students not only have to think about the conditions and conclusions of the question, but also to think what the nature of the mathematics problem is, why we have to adopt the chosen method to solve the mathematical problems, how to find the solution of similar problems and how to overcome the difficulties etc..

Fourthly, mathematics teachers should urge students to be diligent in solving mathematics problems and do more exercise about mathematics problem solving. On this basis, to sum up and reflect on. Only by practice and reflection can we really understand the tacit knowledge used in the mathematics problem solving and accurately grasp them. Polanyi (2000) had pointed out that imitation, practice and reflection were the best ways for students to learn tacit knowledge, especially in the field of scientific research (Xiao, 1999).

Fifthly, mathematics teachers should encourage students to communicate with students or friends in the course of mathematical problems solving. Through their own explanation and demonstration of friends to understand the use of tacit knowledge can more clearly understand the effect of tacit knowledge and more firmly grasp the relevant tacit knowledge.

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