The Research on Teaching of Mathematical Understanding in China

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ABSTRACT

In mathematics teaching, understanding is the value embodiment of the whole mathematics classroom teaching. In recent years, some researchers have conducted extensive discussions on the basic problems of how to teach mathematical understanding from the aspects of theory and practice from different perspectives and have obtained many research results. In order to better carry out the teaching of mathematical understanding, this paper reviews the related literature from the aspects of classroom communication, teaching evaluation, teaching reflection, arrangement of teaching knowledge and materials, arrangement of teaching activities and design of teaching media, hoping to promote the future research and development of mathematical understanding teaching.

Keywords: Mathematical understanding, Mathematical teaching, Mathematical learning.

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1. INTRODUCTION

Mathematical understanding refers to the individual's psychological representation of new knowledge based on existing mathematical knowledge and experience, constructing the personal psychological meaning of new knowledge, constantly improving and developing the mathematical knowledge network in the mind, and can be incorporated into the knowledge network (Jin, 2014). With the deepening of the reform of mathematical teaching, the pursuit of students' substantial understanding of mathematics has become one of the direct objectives of mathematics education and the research on mathematical understanding teaching is in the ascendant. Mathematical understanding is the central link in the process of students' mathematical learning and its research is of great enlightening significance and guiding value to enrich the relevant theories of mathematics pedagogy. In order to promote and overall grasp how to carry out the teaching research of mathematical understanding, we have done the research summary of this aspect.

2. LITERATURE COLLATION METHOD AND PROCESS

The key words of this literature review are mathematical understanding and understanding teaching. The author searched 320 related articles in CNKI and Wan fang database, among which 299 articles were selectively deleted. The reason for deletion is that the article is not a research article or research content that has nothing to do with teaching how to understand math. The remaining 21 papers are basically theoretical studies on the connotation, level, characteristics, factors, obstacles and teaching strategies of mathematical understanding and a very few of them are studied through experimental investigation.

3. COMMUNICATION IN CLASS

Classroom communication refers to the process of teaching information transmission and feedback in classroom situations. It includes communication between teachers and students, students, and teaching materials. It is also the communication of cognition, emotion, intelligence and other aspects, mainly through verbal, voice, physical or situational communication and non-verbal communication (Zhong, 2004). At present, many scholars believe that mathematics classroom communication can effectively promote the teaching of mathematical understanding. Therefore, many people have paid attention to this aspect. Among them, Su Wenjuan believes that teachers should be good at stimulating students to think positively, enabling them to acquire the understanding of things and phenomena, guiding students to express their opinions, encouraging students to express in their own language and solve problems in their own way. In addition, teachers should also be good at asking some questions, allowing students to reflect on their own understanding and understanding of the process. At the same time, classroom communication is not limited to the question and answer between teachers and students, but also includes the interaction between students and students. If the atmosphere of communication is relaxed and equal, cognitive conflicts will easily occur among students. They put forward their own understanding from different angles and draw on the opinions of others, that is, one idea inspires another, so that their thinking can be expanded and their understanding ability can be developed (Su, 2006).

Wang Aizhen pointed out that in class communication, students have a process of rerefining, processing and summarizing knowledge, so that they can gain a deeper understanding of knowledge. On the other hand, each student has different perspectives and levels of understanding of the same knowledge. Through communication, they can learn from each other's strengths and make up for each other's weaknesses (Wang, 2004).
4. EVALUATION IN TEACHING

Teaching evaluation refers to the value judgment of teaching process and teaching results by using scientific and feasible evaluation methods in accordance with certain teaching objectives. It can provide reliable information and scientific basis for improving teaching quality (Wang and Qi, 2014). Through the analysis of the literature, we find that many scholars believe that teaching evaluation can effectively promote the teaching of mathematical understanding. Therefore, many people have conducted research on this aspect, among which Kuang Jinlong and Bao Jingjuan put forward three aspects that should be paid attention to. (1) Teachers should focus on the pertinence of evaluation. For the mathematics content that should be understood persistently, the evaluation strategy adopted should be positioned as a kind of challenging practice activity and students should be given the opportunity to solve the puzzles in the activity. For the mathematical knowledge that should be known and understood, both practical activities and traditional examinations can be used. For the mathematics courses that should be familiar, it can be directly tested in the school's regular paper and pencil test. (2) Teachers should pay attention to the procedural nature of evaluation, and evaluate and promote students' understanding through the design of variants, serialization design and practical activities of problem situations. At the same time, students should be able to express their opinions, methods, understandings, etc. Students' responses should be considered as an important resource for learning and teaching and the resources should be shared by all students in the classroom. (3) Teachers should pay attention to the diversity of evaluation. Evaluation should be mainly divided into formal evaluation and informal evaluation. Informal evaluations include observations of student activities and behaviors to understand students' understanding of mathematical concepts, thinking processes and problem-solving strategies. Information obtained through informal evaluation can improve teaching strategies. Formal evaluation should allow students to show what they know and what they will do, rather than confirm what they don't know (Kuang and Bao, 2013).

Su Wenjuan believes that teachers should choose the problem that suits the current level of understanding of students. Teachers should combine their own teaching design and set up some problems that are closely related to new knowledge, various forms and moderate difficulty. Then they can not only judge their understanding from the students' reactions, but also make students answer questions and solve problems. In the process of further deepening the understanding of the new knowledge (Su, 2006).

5. REFLECTION IN TEACHING

Teaching reflection refers to the active, continuous, thoughtful, in-depth and self-regulated reflection on the teaching activities that have taken place or are taking place, as well as the theories and hypotheses behind these teaching activities, with the support of teachers' tendency to reflect on teaching in order to achieve effective teaching (Shen, 2006). Only through reflection can we better understand the inadequacy, so many scholars believe that reflection has a very important position in the teaching of mathematical understanding. For this reason, scholars are currently paying attention to this aspect and have their own research insights. Among them, Chen Qiong and Weng Kaiqing proposed that teachers should urge students to reflect on their own mathematics learning activities in their teaching, how they think, what mathematical ideas, mathematical methods and mathematical skills are used. It aims to explore the trajectory of the predecessors and themselves, to further understand the essence of mathematical theory, to understand the essence of mathematical thoughts, to understand the true meaning of mathematics, in order to achieve a high level of understanding (Chen and Weng, 2003).

Li Shuwen and Zhang Tongjun pointed out that students should be given the opportunity and time to reflect. According to the "super-regression" mathematical understanding model, when a learner faces a problem that cannot be immediately understood or solved, in order to deepen and expand his understanding, he cannot simply
abstract his understanding into a higher level of understanding. It should be done by returning to the inner level of understanding. Teachers should give students the initiative to learn and eager to seek success. Teachers should allow students to have repeated learning, so that they have time and opportunity to reflect on their own thinking activities and improve their understanding of new knowledge through observation, reflection, abstraction and improvement (Li and Zhang, 2002).

6. ARRANGEMENT OF TEACHING KNOWLEDGE AND MATERIALS

We all know that the understanding of mathematics is students' understanding of knowledge. Therefore, the design of knowledge in teaching is the most important part. The teaching knowledge and material arrangement play a key role in understanding. Among them, Wang Ruilin and Qi Chunxia proposed how to teach mathematical understanding from two aspects. The first is to identify the meaning of each element and break through the understanding of representation. Mathematical objects encountered in mathematics learning often have many elements, and students' mastery of these basic elements is the prerequisite for understanding knowledge. Students' understanding of these mathematical elements can not stay in recognition, but rise to "identification". The second is to reveal the inner relationship through regression and establish the connection correctly. On the one hand, the teacher trace back to the initial position to better grasps the essence of the mathematical object and better abstracts from the lower level to the higher level. on the other hand, it should be traced back to the core concept and the core method, because the core concepts and methods are the fixation point of mathematics learning and the understanding of this knowledge can affect the whole mathematics learning (Wang and Qi, 2014).Liu Lianghua proposed strategies to promote mathematical understanding teaching from four aspects: (1) Providing rich sensible materials. By providing students with rich and sensible materials sufficient to explain relevant knowledge, teachers can use them to carry out various complex cognitive activities and establish the characteristics, perceptions, representations or concepts in the mind to understand the things, so that the students can obtain specific or perceptual knowledge of things. (2) Mathematical knowledge should be arranged in a spiral way. In the process of mathematical teaching, the basic concepts and principles should be taken as the core and the knowledge should be arranged in a spiral way so that students can repeatedly contact important basic concepts and principles until they have mastered the knowledge system adapted to these basic concepts. (3) Strengthen the structure and systematization of mathematical knowledge. First of all, the main concepts and knowledge learned are sorted out and summarized in a small scope. Then, the important knowledge is reorganized and organized in a larger scope, the logical order between the knowledge is clarified and a knowledge network with rich connections is constructed in the brain. (4) Teachers should implement variant teaching so as to deepen students' understanding of the mathematical knowledge they have learned and make all the points of knowledge integrated and coherent. (Li and Zhang, 2002; Liu, 2009.). Guo Penggui and Chen Dunying believe that (1) Supplement the corresponding declarative knowledge. When teaching a new lesson, teachers must first define what preparatory knowledge is needed for new knowledge. The review before the new lesson is not only the content of the previous lesson, but should include all the preparatory knowledge necessary for the new knowledge. (2) To help students choose a commensurate cognitive pattern, the teacher should select the teaching strategy that is most conducive to the student's mobilization of the cognitive schema, so that students can master the corresponding procedural knowledge. (3) Mathematical scenarios showing mathematics knowledge. In the study of mathematics knowledge, teachers provide students with rich and sensible materials sufficient to explain relevant knowledge, so that students can carry out various complex cognitive activities and establish them in their minds. Recognize the characteristics, perceptions, representations or ideas of things to gain some specific or perceptual knowledge of things (Guo and
Zhang Qian believes that (1) To activate the growth point of new knowledge, teachers should understand what students have learned, done, seen, heard and on this basis to establish an appropriate fulcrum. (2) To grasp the importance and difficulty of new knowledge, the primary task of teachers in teaching is to reveal the scientific connotation of key content so that students can understand the nature of knowledge; to guide students to analyze and integrate the internal relations between new knowledge, the differences between new knowledge and old knowledge and to help students master the nature of knowledge; to provide opportunities to experience the use of knowledge. (3) Transferring knowledge points. In the variant, we should pay attention to the gradient of the problem. The span can not be too large and can not be fixed in situ (Zhang, 2013).

Wang Aizhen and Liu Guangjun believed that the essence of knowledge should be revealed through positive and negative discriminating teaching methods, thus enhancing students' accuracy and profoundness in knowledge understanding. For example, within the cognitive level of students, teachers can give some counterexamples and special cases and conduct positive and negative comparisons and analysis, so that students can identify the connections and differences of related objects, thus enhancing understanding (Liu, 2000; Wang, 2004).

7. DESIGN OF TEACHING ACTIVITIES

Teaching activities refer to the educational activities in which teachers impart objective knowledge, exercise skills, enlighten wisdom, guide correct value realization and stimulate positive emotional experience to the students through appropriate teaching methods and teaching contents. Research on teaching activities plays an important role in promoting the teaching of mathematical understanding and many scholars have paid close attention to it. Lu Xiaqing put forward two suggestions: (1) Create rich situations and design learning tasks that promote thinking. When designing learning tasks, teachers should present various forms. Let students "do their best" with loose and open activities and allow diversified answers instead of only understanding. (2) Experience the process of “re-creation” and let students experience the formation of mathematical thinking methods. Learning is not pure imitation and memory. It should provide students with the time and opportunity to explore knowledge through reasonable mathematical activities and enable them to experience the "re-creation" process of knowledge (Lu, 2012). Yang Huiqing proposed that teachers should create "internships" to provide students with opportunities to participate in practice. This allows students to experience the process of mathematics creation, stimulate students' interest and motivation, cultivate students' problem solving and practical ability, improve students' understanding and form a correct mathematical view (Yang, 2009). Yang Hongjian believed that teachers should pay attention to the life of mathematics knowledge. Mathematics is closely related to real life, so it is necessary for teachers to guide students to practice reasonably in connection with the actual life of students. This not only can arouse the student's study enthusiasm, but also can fundamentally improve the student's mathematics application ability. By designing practical exercises which are closely linked with real life, we can fully stimulate students' interest in learning (Yang, 2016). Gong Xiaomei proposed to improve students' mathematical understanding through mathematics reading activities in classroom teaching (Gong, 2009). Wang Ruilin and Qi Chunxia thought that they should be found in mathematical exploration and stimulate the generation of creativity. Mathematical exploration includes observation, experiment, conjecture, induction, analogy, generalization and a series of forms. In daily teaching, penetrating inquiry will become the foundation of students' creation (Wang and Qi, 2014).

8. DESIGN OF TEACHING MEDIA

Teaching media is the carrier of teaching content, the manifestation of teaching content and the tool of transmitting information between teachers and students. For example, objects, oral language, charts, images and...
animations are teaching media. Teaching media are often realized by certain material means, such as books, blackboard, projectors, videos and computers. It is generally recognized that we should reasonably use the teaching media to promote the teaching of our mathematical understanding. Therefore, many scholars have their own opinions on this aspect. Xue liang believes that mathematical tools should be fully used. With the constant development of information technology, new teaching tools are encouraged in the process of teaching. Teachers can use multimedia, video and audio to help with teaching. When integrating some mathematics knowledge, teachers can use multimedia to design and use different colors and sizes to classify, which will make it easier for students to understand and remember. For example, we can use multimedia to display the three views and three-dimensional graphics in geometry study, so that the spatial stereoscopic impression of the graphics will be stronger (Xue). Lu Wenjuan proposed: (1) Using relevant software tools to properly sort out and summarize mathematical knowledge, so that students can not only understand the mathematical knowledge representation, but also deepen the essential understanding of the internal relationship and difference between knowledge. (2) To guide students to participate in online group collaboration and discussion activities outside of class. Guide students to use online tools such as e-mail, chat tool QQ and online community to conduct group communication and exchange ideas on learning topics (Lu, 2012).

9. EVALUATION OF EXISTING RESEARCHES

In summary, it can be seen that the current research on how to conduct mathematical understanding is mainly focused on the teaching of primary and secondary schools. These studies mainly analyze the teaching knowledge, the design of situational activities, teaching reflection, communication and evaluation and teaching media. The coverage is wider and the conclusions are more comprehensive. Some studies rely on mathematical models to analyze their problems and countermeasures and some studies are conducted by teaching front-line personnel in combination with teaching practice, which is worthy of advocacy.

However, from the above analysis, we can see that there are also some shortcomings in this study mainly reflected in the following two points:

(1) Research content needs to be expanded.

The current research only focuses on the teaching of mathematics understanding in primary and secondary schools, while ignoring the existing difficulties in teaching mathematics in higher education institutions and vocational colleges. This part has a large gap and the research object is not comprehensive enough. At the same time, we should note that many students are less suitable for the leap of knowledge when they are in the process of progression, but there are few studies in this area.

(2) Research method remains to be enriched.

Most of the research reviews are based on theoretical analysis, with the help of relevant theories of psychology and pedagogy. Few practical researches are carried out by means of investigation or experiment from the point of view of teaching practice, which inevitably makes these research results not well applied to teaching practice.

Therefore, it is necessary to supplement the future research from the research content and pay more attention to the higher education, the field of vocational education and how to carry out the teaching of mathematical understanding in the convergence of higher education and higher education. According to the different characteristics of different objects, specific problems are analyzed and corresponding strategies are obtained, thus
enriching the whole teaching system of mathematical understanding. At the same time, we should perfect the research methods, adopt more scientific methods of experiment or investigation, combine with reality and devote ourselves to practice. Only in this way can the conclusions be more scientific and convincing and the achievements can be better applied to our teaching.

REFERENCES


