Methods Used by Fourth Graders when Responding to Number Sense-Related Questions

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ABSTRACT

This study examined the methods used by fourth graders when answering number sense-related questions. Eighth fourth graders who were classified as high, medium, and low achievers in mathematics were recruited from three public elementary schools in south Taiwan. Semi-structured interviews were conducted with all students, in which they responded to a series of number sense-related questions. The findings showed that all students tended to use rule-based methods to solve the questions. However, low and medium achievers were more inclined than high achievers to use rule-based methods to solve the problems. Additionally, the findings suggest that students' thinking is limited to memorizing the methods they have learned in schools; thus, they cannot use meaningful approaches to solve problems.

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1. INTRODUCTION

Number sense (NS) has been highly emphasized as a key topic in mathematics education in many developed countries (e.g., Australia, Germany, the United States, and the United Kingdom). This is because traditional mathematics teaching overemphasizes written computation, which limits students' thinking and reasoning ability (Australian Education Council, 1991; McIntosh *et al.*, 1992; Sowder, 1992; Reys and Yang, 1998; National Council of Teachers of Mathematics, 2000; Verschaffel *et al.*, 2007). Moreover, NS plays a key role in everyday situations, as it enables people to solve numerical problems flexibly and efficiently (McIntosh *et al.*, 1992; Verschaffel *et al.*, 2007).

Taiwanese students generally outperform their peers on international mathematics assessments such as The Programme for International Student Assessment [PISA] or Trends in International Mathematics and Science Study [TIMSS] (Mullis *et al.*, 2016; Organization for Economic Co-operation & Development (OECD), 2016). However, students who perform well on these assessments or those are highly skilled in written computation do not necessarily have strong NS. For example, research has shown that Taiwanese sixth and eighth graders who are skilled in written computation do not necessarily develop NS (Reys and Yang, 1998; Yang, 2005). However, few studies have examined NS in Taiwanese fourth graders. Students usually begin to learn more complex rational numbers, whole numbers, and operations in the fourth grade, as it is important for them to acquire basic knowledge of mathematics to support their future learning. This study therefore examined the methods used by fourth graders when answering NS-related questions. The research questions were as follows:

1) What types of methods do fourth graders use when responding to NS-related questions?
2) Do low, medium, and high achievers differ in the methods used to solve NS-related questions?

2. BACKGROUND

2.1. Number Sense Components

In the twenty-first century, NS is considered a key component of the primary and middle grade mathematics curriculum (NCTM, 2000). Defining NS is important. NS can be interpreted as an individual who has a strong intuitive feeling for numbers and a profound understanding of numbers and operations (McIntosh *et al.*, 1992; Markovits and Sowder, 1994; McIntosh *et al.*, 1997; Reys and Yang, 1998) Although researchers have provided different definitions for NS (Berch, 2005) most mathematics educators and researchers (McIntosh *et al.*, 1992; Markovits and Sowder, 1994; Menon, 2004) agree that NS for fourth graders should include the following components:

1) Understanding the basic meaning of numbers and operations;
2) Being able to compose and decompose numbers;
3) Being able to judge whether a computational result is reasonable;
4) Recognizing number size

Having reviewed fourth-grade mathematics textbooks in Taiwan, the four components match the topics and contents discussed in the textbooks. Therefore, the interview questions used in this study were based on the four NS components.

2.2. Number Sense-Related Studies

Do students who have strong computational skills develop strong NS? The answer seems to be negative. For example, Reys and Yang (1998) found that students, regardless of whether they were low, medium, or high achievers, were very accustomed to using standard written computation to solve NS-related questions. They rarely used NS-based methods such as benchmarks, estimation strategies, and other methods (Reys and Yang, 1998).

For example, when students were asked how to solve $\frac{11}{12}+\frac{8}{9}$, most tended to use a paper-and-pencil method. It was very difficult for students to develop and apply 1 as a benchmark to estimate that $\frac{11}{12}$ and $\frac{8}{9}$ are both less than 1, and that the answer should therefore be smaller than 2 (Markovits and Sowder, 1994; Reys and Yang, 1998; Menon, 2004).

Studies of traditional mathematics teaching have shown that it is not only students who tend to use paper-and-pencil computation to solve problems, but also pre-service teachers and in-service teachers who prefer to use rule-based methods (R-based methods) to solve NS-related questions (Markovits and Sowder, 1994; Menon, 2004; Yang, 2005; Yang *et al.*, 2009). Students are skilled at solving numerical problems using the paper-and-pencil method because teaching in Taiwan focuses heavily on standard written algorithms (Reys and Yang, 1998). However, obtaining the correct answer does not imply that students understand the meaning of the question (Reys and Yang, 1998). Thus, although students may be adept at written computation skills and arrive quickly at the correct answer, they often do not really know what they are doing (McIntosh *et al.*, 1992; Yang, 2005).

Research has also shown that Taiwanese fifth, sixth, and eighth graders perform poorly on NS tests (Reys and Yang, 1998; Yang, 2005). Moreover, they tend to use paper-and-pencil based algorithms to solve NS-related questions. We therefore aimed to determine whether fourth graders in Taiwan use similar methods when responding to such questions.

3. METHOD

3.1. Sample

Eight fourth grade students from public elementary schools in southern Taiwan volunteered to be interviewed. Based on school mathematics achievement scores, four students were classified as high achievers (top 10%) [H1, H2, H3, and H4], two as medium achievers (between 40% and 70%) [M1 and M2], and two as low achievers (bottom 20%) [L1 and L2]. The schools are situated in diverse areas; hence, these children came from families reflecting a wide range of occupations, incomes, and educational levels.

3.2. Instrument

Based on earlier studies (McIntosh *et al.*, 1992; Sowder, 1992; NCTM, 2000; Menon, 2004) the NS framework used for the interview comprised four components: 1) Understanding the basic meaning of numbers and operations; (2) Recognizing number size; (3) Being able to compose and decompose numbers; (4) Being able to judge whether a computational result is reasonable. A total of 12 interview questions were used, with three questions for each component. These questions were reviewed by two elementary school teachers and one mathematics educator; they agreed that the questions reflected the meaning of NS, and that the content was appropriate for fourth graders.

3.3. Data Collection

Each student was interviewed separately in a quiet room. They were given a booklet containing the 12 interview questions, and each question was also presented separately on an A4 paper during the interview. Each interview lasted from 30 to 40 minutes and was audio- and-video recorded. The data were then transcribed for analysis.

3.4. Analysis

Students' responses were examined and then carefully categorized. According to the analytical methods used in previous studies (e.g., (Markovits and Sowder, 1994; Reys and Yang, 1998; Yang, 2005)) each response (whether correct or incorrect) was coded according to one of the following three categories:

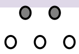
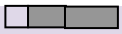
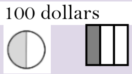
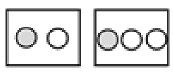
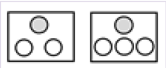
1) NS-based: these methods utilized one or more of the components of NS (i.e., basic meaning of numbers, benchmarks, number magnitude, relative effect of operations on numbers, estimation, and ability to judge reasonableness).
2) Rule-based: these strategies applied the rules of standard written algorithms, but anything beyond the direct application of the rule was unexplained.
3) Could not explain: despite probing and querying by the interviewer, the student was unable to give a clear explanation of how the answer had been obtained.

4. RESULTS

4.1. Methods Used by Fourth Graders When Responding to Number Sense-Related Questions

Table 1 illustrates how fourth graders responded to the NS component: Understanding the basic meaning of numbers and operations.

Table-1. How fourth graders responded in terms of understanding the basic meaning of numbers and operations

	Questions	Methods	Low	Middle	High
1.	Which of the following choices best represents "The shaded area is $\frac{2}{3}$ of the whole"?	(1) 	0	0	0
		(2) 	0	0	0
		(3) 	0	0	0
		*(4) 	2	2	4
		NS-based	(0)	(0)	(2)
		R-based	(0)	(0)	(0)
		Wrong Explanations	(2)	(2)	(2)
2.	An elder brother's bank account contains the smallest possible four-digit number, and his younger brother's bank account has the largest possible three-digit number. What is the numerical difference between their bank accounts?	*(1) 1 dollar	1	1	3
		NS-based	(0)	(0)	(0)
		R-based	(1)	(1)	(3)
		Wrong Explanations	(0)	(0)	(0)
		(2) 2 dollars	0	1	1
		(3) 10 dollars	0	0	0
		(4) 100 dollars	1	0	0
3.	Which of the following figures best illustrates a comparison between $\frac{1}{2}$ and $\frac{1}{3}$?	(1) 	2	0	0
		*(2) 	0	1	3
		NS-based	(0)	(1)	(3)
		R-based	(0)	(0)	(0)
		Wrong explanations	(0)	(0)	(0)
		(3) 	0	1	1
		(4) 	0	0	0

Note: * indicates the correct answer to the question

The results for question 1 show that all students, regardless of whether they were low, medium, and high achievers, were able to choose the correct answer. However, it was difficult for students to provide a verbal definition when asked to explain the meanings of fractions. For example, R : Why is it two-thirds?

L1 : It looks like two-thirds. I don't know how to explain it.

…

R : Why did you not select (1), (2), or (3)? Can you tell me your reasons?

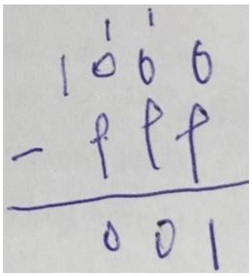
H1 : Because of this ….

…

The results for question 2 show that even though they could all produce the correct answer, none of the students used the NS-based method. All of them tended to apply the R-based method. For example:

R : Can you tell me your reasons?

H2 : Because



R : Can you do it using a different method?

H2 : I don't know.

Three students also selected the incorrect answer. Regardless of achievement level, students' understanding of the place value of whole numbers was very uncertain. For example:

R : The second question? Why did you choose the answer (4)?

L02 : Hmm… (Students were silent for 7 seconds).

R : Why did you choose the answer (4)?

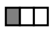
L02 : Well! I don't know.

Question 3 explored students' conceptual understanding of fractions. One student who was a medium achiever and two students who were high achievers selected the correct answer and applied the NS-based method. However, one of the high achievers was able to select the right answer but could not provide the correct explanation. For example:

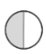
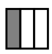
R : Why do you think the answer is (2)?

H01 : Because it is more like the graphs used in my mathematics textbook.

This student stated that the correct answer was selected because option 2 was similar to the pie chart presented in the mathematics textbook. However, the graphs in the other three options were not included in the textbook. Therefore, students may select the correct answer, but their reasoning is incorrect.

Two low achievers selected the incorrect answer. They believed that the answer to this question was   (1) . It is therefore reasonable to conclude that they lack sufficient knowledge of fractions. For example:

R : Why did you select (1)?

L01 : Because this one is 1/2  and that  one is 1/3.

However, one medium achiever and one high achiever selected   (3) as the correct answer. They responded:

R : Why do you want to choose 3?

M02 : Because you can see that two have the same size and it looks fair. This left side box is 1/2 because there are two circles and one is colored. The other box is 1/3 because there are three circles and one is colored.

Based on the aforementioned student's explanation, it is reasonable to believe that students' formation and construction of the concept of fractions were unstable and not meticulous. Some students did not understand the meaning of fractions, and some remembered that the graph shown was similar to the graph used in the textbooks; thus, solving the question did not require their deeper understanding. From the results for questions 1–3, we have found that learning mathematics without any understanding does not help students make sense of the concepts they are learning.

Table-2. How fourth graders responded to questions that involve composing and decomposing numbers

	Questions	Responses		Low	Middle	High
4.	Mother wants to buy a $199 model for her son and a $399 doll for her daughter in a toy store. When she gets to the register, how many $100 bills does she need to take out?	(1)	2	0	0	0
		(2)	4	1	0	0
		(3)	5	0	1	0
		*(4)	6	1	1	4
			NS-based	(0)	(0)	(0)
			R-based	(1)	(1)	(4)
			Wrong explanations	(0)	(0)	(0)
5.	If "134 × 9 = 1206," what is the difference between "134 × 8" and "1206"?	(1)	1	1	1	0
		(2)	8	0	0	0
		*(3)	134	1	1	4
			NS-based	(0)	(0)	(0)
			R-based	(1)	(1)	(4)
			Wrong explanations	(0)	(0)	(0)
		(4)	142	0	0	0
6.	Tom has $248. John has four times as much as Tom's money. Which of the following answers is closest to the sum of their money?	(1)	250	0	0	0
		(2)	1000	1	1	0
		*(3)	1200	1	1	4
			NS-based	(0)	(0)	(0)
			R-based	(1)	(1)	(4)
			Wrong explanations	(0)	(0)	(0)
		(4)	2500	0	0	0

Note: * indicates the correct answer to the question

Table 2 displays the fourth graders' responses to questions that involve composing and decomposing numbers. Question 4 explores whether students can develop and use efficient methods to estimate the answer without using the paper-and-pencil method. The interview results showed that one medium achiever gave the incorrect answer, having incorrectly used a written computational method. However, two low achievers, one medium achiever, and four high achievers gave the correct answer, all using the paper-and-pencil method. No student was able to develop and use an estimation method to solve the question. For example:

R : How did you work out your answer to question 4?

L02 : Because 199 + 399 is 598, so we need six hundreds.

R : Good! Can you do it using a different approach?

L02 : I don't know.

All other students used a similar method to solve this question.

Question 5 explores whether students can apply efficient methods to solve this question without relying on the paper-and-pencil method. The results showed that one medium achiever student gave an incorrect answer, having misapplied a written computational method. However, two low achievers, one medium achiever, and four high achievers all gave the correct answer, having used the paper-and-pencil method. No student was able to develop and use the estimation method to solve the question. For example:

R : How did you work out your answer?

L01 : Well. I need to calculate it using the paper-and-pencil method.

R : Can you do it using a different approach?

L01 : I don’t know.

or

R : How did you work put your answer?

M02 : I calculate 134 × 8 by using the paper-and-pencil method, and then I can do the subtraction.

R : Can you do it using a different approach?

M02 : I don’t know.

Question 6 examines whether students can use a NS-based method and compose and decompose numbers to solve this question. The results showed that all students tried to use the paper-and-pencil method to find the exact answer. For example:

R : How did you work out your answer?

H02 : I needed to use a paper and pencil.

R: R : Can you do it using a different approach?

H02 : I don’t know.

From the results for questions 4–6, we found that it was difficult for students to use the NS-based method to decompose and compose numbers. All students tended to use a written computation method to solve the problem. Students were, however, limited by standard written algorithms; therefore, it was difficult for them to develop and use an efficient method to solve the problem. This result is consistent with earlier findings, which showed that students tended to use standard written computation to solve NS-related questions (Markovits and Sowder, 1994; Reys and Yang, 1998; Menon, 2004).

Question 7 examines students’ ability to judge whether a computational result is reasonable in the context of a real-world situation. The results showed that all students experienced difficulties in judging whether a computational result was reasonable. They all responded, “I can’t compare.” For example:

R : How did you work out your answer to question 7?

H02: 5000 books cannot be put into the bags, two hands cannot be used to lift a pig weighing 5000 g, and no way can anyone fit 5000 candies into the mouth. Unless his face is fat, I can’t find the answer.

During the interview, we also found that students had difficulty making sense of measurement units in real-life situations. For example:

R: Why do you believe that your hands cannot lift a pig weighing 5000 g?

M01: Because 5000 is a very large number.

Most students believed that 5000 is a very large number; therefore, they believed that they could not lift a pig weighing 5000 g.

Question 8 assesses the student's ability to judge whether a computational result related to whole numbers and operations is reasonable. The results showed that one high achiever believed that the sum of 2 three-digit numbers is a 3-digit number. He responded thus:

R : How did you work out your answer to question 8?

H01 : The sum of 2 three-digit numbers should be a 3-digit number, because 123 plus 112 equals 235, which is a 3-digit number.

R : Can you show me some different examples?

H01 : 380 + 250 = 630; it is a 3-digit number.

However, one medium achiever and one high achiever responded that the sum of 2 three-digit numbers is a 6-digit number. For example:

R : Why did you choose 6 digits?

H02 : Because 3 plus 3 equals 6.

R : Are you sure?

H02 : Yes!

Moreover, two low achievers, one medium achiever, and two high achievers provided the correct answer, although no student could provide reasonable explanations as to how they did this. Two low achievers gave the correct answer by guessing, as they responded: "I don't know how to explain. I guess."

Table 3 presents the fourth-grade students' responses to questions asking them to judge whether a computational result is reasonable

Table-3. How fourth grade students responded to questions asking them judge whether a computational result is reasonable

	Questions	Responses	Low	Middle	High
7.	Whose statement is the most reasonable? Joe: "I can fit 5000 textbooks into my backpack." Lin: "I can lift a pig that weighs 5000 g." Kan: "I can fit 5000 M&Ms into my mouth "	(1) Joe	0	0	0
		*(2) Lin	0	0	0
		NS-based	(0)	(0)	(0)
		R-based	(0)	(0)	(0)
		Wrong explanations	(0)	(0)	(0)
		(3) Kan	0	0	0
		(4) Can't compare	2	2	4
8.	How many digits does the sum of 2 three-digit numbers contain?	(1) 3 digits	0	0	1
		(2) four digits	1	0	0
		*(3) 3 or 4 digits	1	1	2
		NS-based	(0)	(0)	(0)
		R-based	(1)	(1)	(2)
		Wrong explanations	(0)	(0)	(0)
		(4) 6 digits	0	1	1
9.	What is the height from the floor to the ceiling of a classroom? Which of the following statements is reasonable?	*(1) 300 cm	0	0	3
		NS-based	(0)	(0)	(0)
		R-based	(0)	(0)	(0)
		Wrong explanations	(0)	(0)	(3)
		(2) 300 millimeters	1	0	0
		(3) 300 m	0	0	1
		(4) Can't compare	1	2	0

Note: * indicates the correct answer to the question

Question 9 examines whether students can use an estimation strategy to estimate the height from the floor to the ceiling of a real classroom. The results showed that three achievers provided the correct answer by using a NS-based method. One low achiever believed that the height should be 300 mm, while one high achiever believed that the height should be 300 m. Moreover, one low achiever and two high achievers could not work out the answer. For example:

R : How did you work out your answer to question 8?

L01 : I don't know how to compare it.

or

R : How did you work out your answer to question 8?

M02 : I don't know.

It is reasonable to conclude that they cannot make sense of measurement units in everyday situations.

The results for questions 7–9 showed that these students lack the ability to judge whether a computational result is reasonable. This may be explained by two factors. First, it is difficult for students to connect the mathematical knowledge they have learned in schools to everyday situations. For example, students can remember that 1 kg is equal to 1000 g, and that 1 m is equal to 100 cm. However, they struggle to understand the meanings of meter, centimeter, kilogram, or gram in real-life situations. This is similar to the findings of earlier studies, in which many students could quickly state that 1 kg = 1000 g or 1 m = 100 cm, but they could not make sense of the everyday meaning of measurement units (Reys and Yang, 1998; Menon, 2004; Yang, 2017). Second, most students tended to use standard written computation to solve problems. Although they were encouraged to use different methods to solve questions, they could not develop alternative methods. The traditional written computation method seems to limit their thinking and reasoning ability. This result is similar to the findings of earlier studies, in that paper-and-pencil algorithms may prevent students from developing different approaches to problem solving (Wearne and Hiebert, 1988; Sowder, 1992; Reys and Yang, 1998; Menon, 2004).

Table-4. presents the results of fourth graders' responses to questions that involve recognizing number size.

Table-4. How fourth graders' respond to questions that involve recognizing number size

	Questions	Responses		Low	Middle	High
10.	A case of Coca Cola has 24 bottles. Bob bought 0.4 of a case and Tim bought $\frac{1}{2}$ of a case. Who bought more?	(1)	Bob	1	1	1
		*(2)	Tim	1	1	3
			NS-based	(0)	(0)	(0)
			R-based	(1)	(1)	(2)
			Wrong explanations	(0)	(0)	(1)
		(3)	Same	0	0	0
		(4)	Can't compare	0	0	0
11.	Which of the two fractions $\frac{5}{7}$ and $\frac{5}{8}$ is larger?	*(1)	5/7	0	1	2
			NS-based	(0)	(0)	(0)
			R-based	(0)	(0)	(0)
			Wrong explanations	(0)	(1)	(2)
		(2)	5/8	2	0	2
		(3)	Same	0	0	0
		(4)	Can't compare	0	1	0
12.	Mom bought two pizzas of the same size. Her son ate $\frac{1}{2}$ of one pizza and her daughter ate $\frac{5}{8}$ of the other pizza. Who ate more?	*(1)	Daughter	2	2	4
			NS-based	(0)	(0)	(0)
			R-based	(1)	(2)	(4)
			Wrong explanations	(1)	(0)	(0)
		(2)	Son	0	0	0
		(3)	Same	0	0	0
		(4)	Can't compare	0	0	0

Note: * indicates the correct answer to the question

Question 10 examines whether a student can develop and use a benchmark to efficiently compare the size of fractions and decimals. The data showed that one low achiever, one medium achiever, and one high achiever gave an incorrect answer. For example:

R : How did you work out your answer to question 10?

L01 : I don't know. I guessed.

Conversely, one low achiever, one medium achiever, and three high achievers gave the correct answer. However, no student could develop and use a benchmark to solve this problem. For example, the low achiever gave the correct answer, but could not explain why. For example:

R : How did you work out your answer to question 10?

L02 : I don't know. I guessed

By contrast, one medium achiever student and two high achievers used a rule-based method to solve this question. For example:

R: How did you work out your answer to question 10?

M02: Because 1/2 = 0.5, so 0.4 < 0.5.

Additionally, one high achiever gave the correct answer, but supported it with an incorrect explanation. This suggests this student misunderstood the concept of fractions. For example:

R : How did you work out your answer to question 10?

H02 : Because Bob bought 0.4 of a case of coca with 24 cans, so Bob only has 4 cans. Tim bought one-half of a case of coca with 24 cans, so Tim has a half of 24 cans.

R : Can you do it using a different method?

H02 : I don't know.

Question 11 examines whether students can use the concept of fractions to compare number size. The results showed that students had difficulty comparing the size of fractions with the same numerator and different denominators. In mathematics classes, fourth graders in Taiwan are taught to compare the size of fractions with the same denominator and different numerators. Therefore, it is a challenge for these students to solve this question. The data also showed that two low achievers and two high achievers gave an incorrect answer. For example:

R : How did you work out your answer to question 11?

L02 : $\frac{5}{8} > \frac{5}{7}$.

R : Why?

L02 : Because 8 > 7, so $\frac{5}{8} > \frac{5}{7}$.

Or…

R : How did you work out your answer to question 11?

H : I don't know.

These students either had misconceptions or guessed when comparing the size of fractions. However, one medium achiever and two high achievers were able to provide the correct answer but could not clearly explain why.

R : How did you work out your answer to question 11?

H02 : 7 + 1 = 8, 5 + 1 = 6, which is more than five-eighths, so 5/7 is relatively large.

R : Why?

H02 : I don't know how to explain it.

Other students had the same problem. This question would not be difficult if students had a profound understanding of fractions. However, they tried to solve the problem by using standard written algorithms. This limits their thinking and problem-solving abilities.

Question 12 examines whether students can develop and use a benchmark to determine the size of fractions. The results showed that all students could select the correct answer; however, one low achiever could not support this with a reasonable explanation, whereas other students used a R-based method to solve this problem. For example:

R : How did you work out your answer to question 11?

L01 : I don't know. I just guessed.

or

R : How did you work out your answer to question 11?

H03 : Because 1/2 is equal to 4/8, so 5/8 > 4/8.

R : Can you use a different method to solve this problem?

H03 : I don't know.

Or

R : How did you work out your answer to question 11?

M02 : Because the sister has eaten five-eighths, and the other side is four-eighths.

Therefore, 5/8 > 4/8

R : Can you use a different method to solve this problem?

H03 : I don't know.

The results for questions 10–12 showed that all students tried to use an R-based method to solve the problems. It seems difficult for students to use $\frac{1}{2}$ as a benchmark and decide quickly and efficiently that $\frac{5}{8}$ is greater than $\frac{1}{2}$. The results of this study are similar to those of previous studies, in which students tended to use a R-based method, and few could develop and apply a benchmark to solve problems (Markovits and Sowder, 1994; Reys and Yang, 1998; Menon, 2004).

Table-5. Frequencies and percentages of correct/incorrect responses to questions 1–12

Responses	Low	Middle	High
Incorrect			
C1	3	2	2
C2	3	3	0
C3	5	5	7
C4	3	2	3
Total	14 (58.3%)	12 (50%)	12 (25%)
Correct			
	NS- R- WRE	NS- R- WRE	NS- R- WRE
C1	0 1 2	1 1 2	5 5 0
C2	0 3 0	0 3 0	0 12 0
C3	0 1 0	0 1 0	0 2 3
C4	0 2 1	0 3 1	0 6 3
Subtotal	0 7 3	1 8 3	5 25 6
Total	10 (41.7%)	12 (50%)	36 (75%)

Note: C1: Understanding the basic meaning of numbers and operations
C2: Being able to compose and decompose numbers;
C3: Being able to judge whether a computational result is reasonable;
C4: Recognizing number size
NS: Number sense-based method; R: Rule-based method; WRE: Wrong Explanation

Table 5 reports the frequencies and percentages of correct and incorrect responses to questions 1–12. The data showed that over half of the low and middle achievers and a quarter of the high achievers could not give correct answers. The results therefore showed the performance of participants responding to NS-related questions to be unsatisfactory. In terms of correct responses, no responses by low and medium achievers and 5 (10.4%) responses by high achievers demonstrated the use of the NS-based method to solve problems. Furthermore, 7 (29.2%) responses by low achievers, 8 (33.3%) responses by medium achievers, and 25 (52.1%) responses by high achievers demonstrated the use of the R-based method to solve problems. The low percentages for the use of the NS-based method and higher percentages for the use of the R-based method revealed that most students lack NS. This finding is similar to earlier studies, in which students performed poorly on tests of NS (Reys and Yang, 1998; Menon, 2004; Yang, 2005).

5. DISCUSSION AND CONCLUSION

This study reported the results of interviews that focused on the methods used by eight fourth grade students from three primary schools in Taiwan. However, because only a few students were interviewed, the study is limited in terms of representativeness; thus, any generalization of the results should be made with caution. Nevertheless, this study provides some important findings, which are as follows:

First, the results of the interview showed that, regardless of the achievement level, students relied heavily on standard written computation methods to solve NS-related questions. Furthermore, very few NS methods, such as using a benchmark to compare number size, decomposing and composing numbers, or judging the reasonableness of a computational result, were used by students to solve NS-related questions. This finding is similar to those of earlier studies, in which children tended to use a R-based method to solve problems, and only a few used a NS-based method (Markovits and Sowder, 1994; Reys and Yang, 1998; Menon, 2004; Yang, 2005). For example, Yang (2005) found that "the overemphasis on standard written algorithms not only discourages children from developing NS, but also hinders the development of thinking and reasoning" (p. 331). Several studies have argued that it will be difficult for students to develop a profound conceptual understanding if their mathematical learning focuses primarily on drill and practice (Hiebert, 1984; Wearne and Hiebert, 1988; Reys and Yang, 1998; Yang, 2005). The teaching and learning of mathematics in Taiwan should therefore focus more on conceptual understanding and the development of NS, and not simply focus on learning procedural knowledge. Second, during the interview, we found that it was difficult for students to apply the mathematical knowledge they learned in school classrooms to solve real-life problems. Students could quickly respond when asked about the relationships between meter and centimeter or between kilogram and gram, but they usually could not make sense of such relationships or apply them to solve problems. This finding is similar to that of previous studies showing that fifth, sixth, and eighth grade students in Taiwan have difficulty using mathematical knowledge to solve realistic problems (Yang, 2005; Yang, 2017). Several earlier studies and reports on mathematics education have argued that emphasizing the connections between mathematical knowledge and real-life situations should be a priority in teaching and learning, thus ensuring that students can apply their learnt mathematical knowledge to real-world problems (Lesh and Lamon, 1992; Van Den, 1996;2001; NCTM, 2000; Mullis *et al.*, 2004). In conclusion, this study found that fourth grade students lack NS. They tend to use standard written computations to solve problems and do not try to make sense of what they are doing. NS has long been considered an important topic in mathematics education (NCTM, 2000; Yang, 2005; Verschaffel *et al.*, 2007) however, few NS-related activities can be found in elementary mathematics textbooks (Yang *et al.*, 2008). To improve fourth grade students' NS, appropriate action should be taken, including the design of appropriate NS activities for students and support for teachers so that they know how to teach NS. We hope that this study will be helpful in redirecting improvement efforts, and that this study provides useful indicators for the future design of the curriculum in Taiwan.

REFERENCES

Australian Education Council, 1991. A national statement on mathematics for Australian schools. Melbourne: Curriculum Corporation.

Berch, D.B., 2005. Making sense of number sense: Implications for children with mathematical disabilities. Journal of Learning Disabilities, 38(4): 333-339. *View at Google Scholar* | *View at Publisher*

Hiebert, J., 1984. Children's mathematics learning: The struggle to link form and understanding. Elementary School Journal, 84(5): 497–513. *View at Google Scholar* | *View at Publisher*

Lesh, R. and S.J. Lamon, 1992. Assessment of authentic performance in school mathematics. Washington, DC: American Association for the Advancement of Science.

Markovits, Z. and J.T. Sowder, 1994. Developing number sense: An intervention study in grade 7. Journal for Research in Mathematics Education, 25(1): 4-29. *View at Google Scholar* | *View at Publisher*

McIntosh, A., B.J. Reys and R.E. Reys, 1992. A proposed framework for examining basic number sense. For the Learning of Mathematics, 12(3): 2–44. *View at Google Scholar*

McIntosh, A., B.J. Reys, R.E. Reys, J. Bana and B. Farrell, 1997. Number sense in school mathematics: Student performance in four countries. Perth: Edith Cowan University.

Menon, R., 2004. Elementary school children's number sense. International Journal for Mathematics Teaching and Learning, 57: 1-16. *View at Google Scholar*

Mullis, I.V.S., M.O. Martin, S. Goh and K. Cotter, 2016. TIMSS 2015 encyclopedia: Education policy and curriculum in mathematics and science. Available from Boston College, TIMSS & PIRLS International Study Center http://timssandpirls.bc.edu/timss2015/encyclopedia/ [Accessed December 12, 2016].

Mullis, I.V.S., M.O. Martin, E.J. Gonzalez and S.J. Chrostowski, 2004. TIMSS 2003 international mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades. Chestnut Hill, MA: Boston College.

National Council of Teachers of Mathematics, 2000. The principles and standards for school mathematics. Reston, VA: NCTM.

Organization for Economic Co-operation & Development (OECD), 2016. Pisa 2015 results: Excellence and equality in education. Paris: Author, 1.

Reys, R.E. and D.C. Yang, 1998. Relationship between computational performance and number sense among sixth- and eighth grade students in Taiwan. Journal for Research in Mathematics Education, 29(2): 225–237. *View at Google Scholar* | *View at Publisher*

Sowder, J., 1992. Estimation and number sense. In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning. New York: Macmillan. pp: 371–389.

Van Den, H.-P.M., 1996;2001. Assessment and realistic mathematics. Utrecht, the Netherlands: Freudenthal Institute, University of Utrecht.

Verschaffel, L., B. Greer and E. De Corte, 2007. Whole number concepts and operations. In F. K. Lester, Jr. (Ed.), Second handbook of research on mathematics teaching and learning. Charlotte, NC: Information Age. pp: 557–628.

Wearne, D. and J. Hiebert, 1988. Constructing and using meaning for mathematical symbols. The case of decimal fractions. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades. Reston, VA: National Council of Teachers of Mathematics. pp: 220–235.

Yang, D.C., 2005. Number sense strategies used by sixth grade students in Taiwan. Educational Studies, 31(3): 317-334. *View at Google Scholar* | *View at Publisher*

Yang, D.C., 2017. Performance of fourth graders when judging the reasonableness of a computational result. International Journal of Science and Mathematics Education: 1–19. *View at Google Scholar* | *View at Publisher*

Yang, D.C., M.N. Li and C.I. Lin, 2008. A study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. International Journal of Science and Mathematics Education, 6(4): 789-807. *View at Google Scholar* | *View at Publisher*

Yang, D.C., R.E. Reys and B.J. Reys, 2009. Number sense strategies used by pre-service teachers in Taiwan. International Journal of Science and Mathematics Education, 7(2): 383-403. *View at Google Scholar* | *View at Publisher*